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CONTENTS

Photos: Cover, Section of model by Laboratorio Central of structure by Prof. Horacio Caminos, failure at lower left; pp. 2, 6, 8, 12, Laboratorio Central, Madrid; p. 23, Prof. Haas; Pp. 26, 34, Cubiertas Ala; p. 38, A. Cogswell; pp. 46, 54, 57, back cover, Synergetics, Inc., illustrations pp. 48, 49, 50, 51, 53, Prof. Duncan R. Stuart.

INTRODUCTION

CHARLES H. KAHN, *engineer and architect, professor in the School of Design, N. C. State College, Raleigh, N. C. has worked in Europe in the field of model testing.*

To even the most conservative among those who see in architecture one of the prime art media, the past twenty-five years must have been an extremely stimulating period. It has been a period of active participation in architectural progress of a large number of giants in the realm of style development. It is quite paradoxical that in this period of rapid growth and strong individual leadership there has flourished and continues to flourish a vibrant stylistic movement without any indication of one strong leading hand.

This movement is characterized by the search for form: the doubly-curved surface, the membrane, the reflection of structural principles in shape generation. To see the Palazetto dello Sport by Nervi, the Madrid Hippodrome by Torroja, one of the simple concrete shells of double curvature by Candela is to realize that in these cases there exists a unity of architectural, structural and constructional behavior which is valid from all the standpoints of possible criticism. These are conceptions in which the final product is a true representation of the design problem.

With the excitement generated by this experimentation, however, have also arisen symptoms of degeneracy, even in the beginning years, which give rise to serious misgivings as to the validity of a great many structures which comprise the nucleus of accepted important examples within the field of this movement. This uneasiness is a result, in many cases, of a trend on the part of the architect to strive for form in structure for the sake of novelty alone, neglecting the structural principles behind the conception and often even the architectural reason for the choice.

It is compounded by the engineering profession which evidences symptoms of an almost complete inability to communicate with the architect, a slavish dedication to analysis procedures which are not necessarily applicable to contemporary systems, and an inability to talk in terms other than the specifics represented by easily calculable answers. Too few engineers have the desire or the training to suggest to the architect modifications of form in accordance with the principles of statical behavior. The engineer has become too willing either to reject totally a new conception or to design anything presented to him without modification.

This has resulted in a curious phenomenon. With a few exceptions, the engineer has become an adjunct of the mechanics of producing the physical structure and has abdicated to the architect the functions of creative engineering and form development. The architect, on his part, with usually less than an adequate knowledge of structural behavior beyond the simple systems and an insufficient background of classical training to have developed for himself some approach to what has been called "intuitive statics", has become a decorator, often insisting on the concrete realization of a form in a structure even after its structural characteristics have been questioned.

There have resulted a whole group of buildings which are seriously deficient from both the economical and the functional point of view but which, to the first glance, seem to embody those principles inherent to the revolution in form which characterizes the contemporary structural approach to architecture. Because of the fact that these buildings lack truly critical examination by the members of the design professions and by the architectural publications of this country, the aura of approval of their stylistic characteristics as "in the contemporary vein" is carried over to the consideration of their economic and functional aspects.

Some of the creators of these structures, when pressed, will admit that the conception was driven to a realization in prototype form, in spite of the doubt raised, because of an attachment to the conception strictly from the standpoint of the plastic design. All too many architects, however, try to justify their buildings from criticism of structural behavior by applying an artificial rationalization which is, at best, extremely dubious. Even more seem to feel that one of the criteria for success in a structure is the inclusion of one or more of the accepted forms of contemporary architecture (e.g. the folded plate, the hyperbolic paraboloid etc.) regardless of the applicability of these structural forms to the problem at hand.

Where does the architect turn who is interested in the new forms available and who is flexible enough in his mind to modify the product of his first inspiration to seek compatibility with structural realities or, in the last resort, to reject the first inspiration when it proves to be possible only by erecting two structures: one, the true structure, to support the building; the other, the accoutrements of structure which will give the impression of a certain behavior but which in reality are mere window dressing?

I believe the solution lies in a correct evaluation of the paths which are open to the designer, be he architect or engineer, and to the collaborating engineer. Basically these paths are mathematical analysis and model testing. Neither is totally sufficient without the other when we deal in the realm of contemporary forms. All too often, the engineer tends to minimize the advantages of model analysis work for prototype structures. Even more often, the architect seems to feel that the model presents the opportunity of complete design within itself.

There exists a group of designers, somewhat defying the classification of architect or engineer, who have reached a state of balance between the two avenues of design approach. They have reached this enviable position through the combination of a rigorous background in the statical behavior of structures and the development of a sixth sense as far as structures are concerned which I have chosen to call "intuitive statics."

Among these men are Professors Torroja, Candela, Benito, Haas, Nervi, Pizzetti and Oberti. At one time or another in the School of Design Magazine articles have been published reflecting the philosophies of these men. In this issue, this investigation into the relationship of architecture and structure is continued with articles by Professor Benito from Madrid; Professor Haas from Delft; Professor Uyanik from Raleigh; and Candela from Mexico City.

It is hoped that this continuing deep investigation will result in a broader area of contact and understanding between the architect and the engineer. It is only through this hope of a vital collaboration between the two contemporary counterparts of the old master builder that the revolution in form which is one of the major movements in contemporary architecture can be rescued from the danger of an early degeneration.

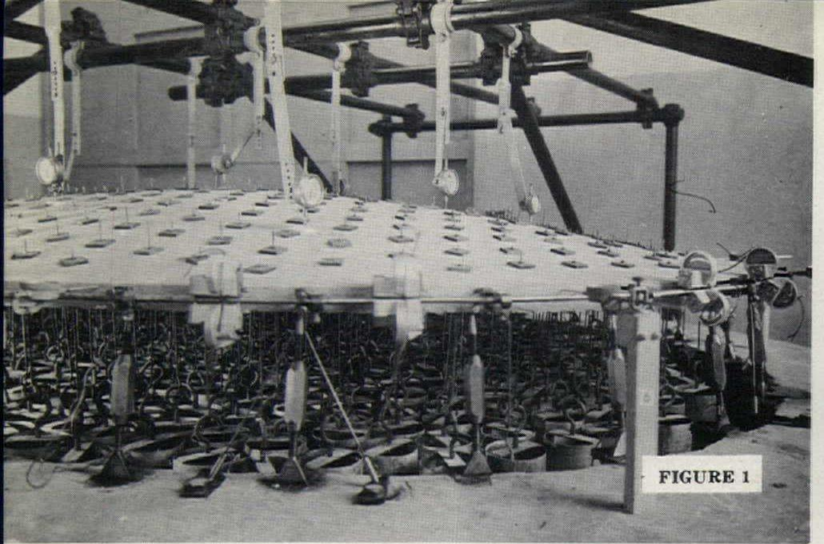
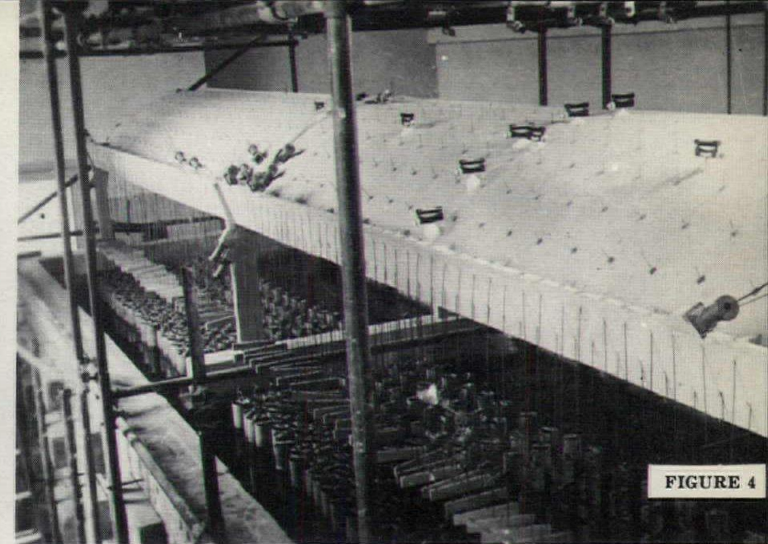


FIGURE 1



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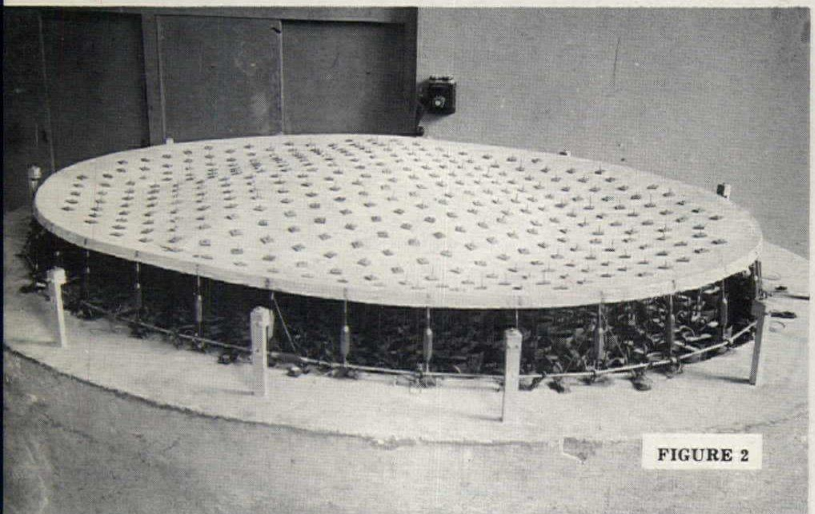

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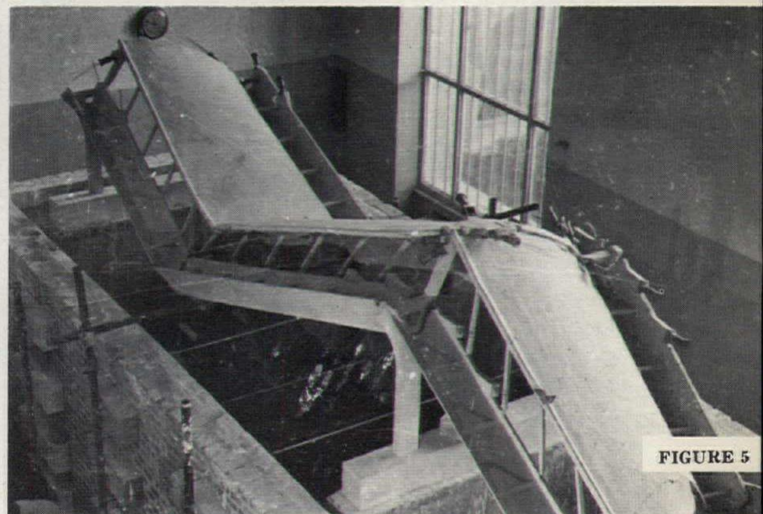

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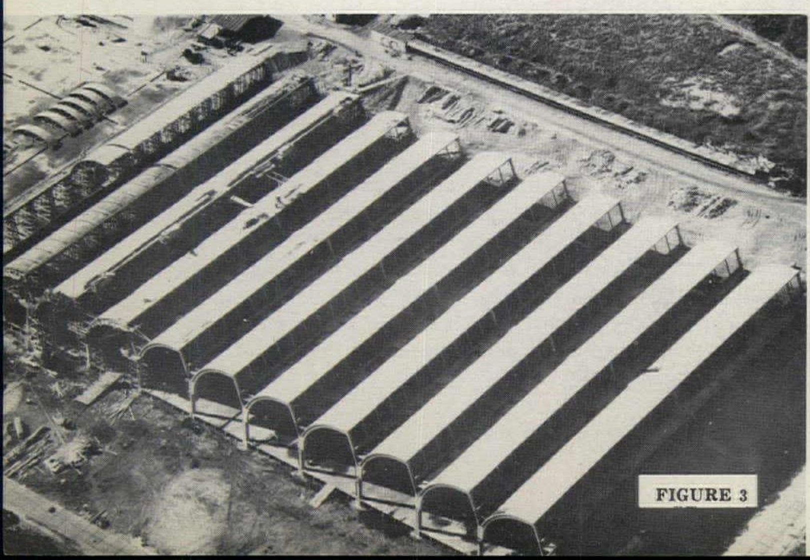

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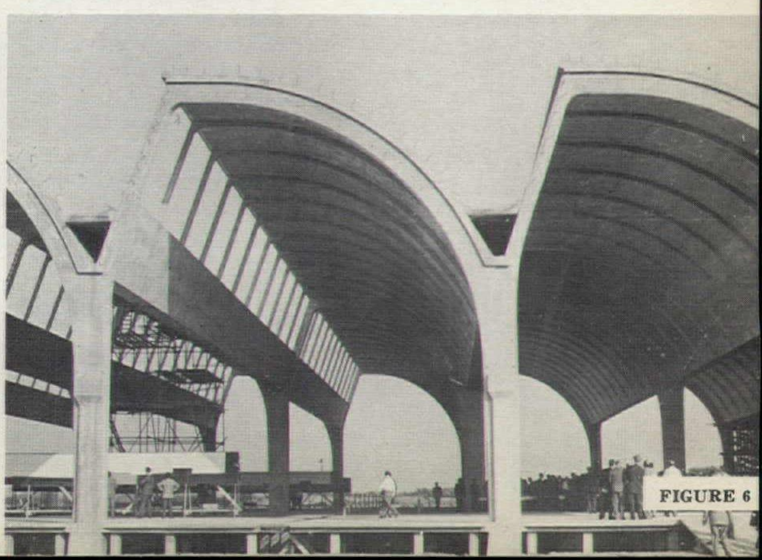


FIGURE 6

EXPERIMENTAL TESTING OF THIN SHELLS BY MEANS OF REDUCED SCALE MODELS

EDUARDO TORROJA *is widely known for his work with concrete thin shells and for the work of the Laboratorio Central de Madrid, of which he is Director.*

The present tendency to give imagination a free reign in the creation of new architectural forms, utilizing the enormous possibilities offered by new materials, requires, on the one hand, complete revision of the classical esthetic canons and, on the other, new methods of verifying structural forms applicable to such new multiple and complex spatial forms that they make inapplicable current theories of strength of materials and planar elasticity. Mathematics in its traditional forms of application is ineffective and even modern electronic computers find with difficulty practical and economic application to the solution of these pròblems in the form that the designer needs for his approximate calculation and to guide his imagination.

Because of the above, the techniques of experimental analysis of stress conditions in many structural types and especially in shells are becoming constantly more important. These are difficult and delicate techniques of which Professor Benito is a particularly masterful specialist. He has been dedicating himself to their perfection for many years with special resolution and constancy. There is none better than he to teach the designer the possibilities of these techniques and the utility which they have for a creative imagination. The latter in architectural problems must always be accompanied by certain numerical theoretical knowledge; the designer need not know the technique thoroughly, but he does need to know what is offered to control the embryos of that imagination.

Translated by Stanley T. Ballenger, Associate Professor of Modern Languages, North Carolina State College, at the request of the School of Design, N. C. State College.

CARLOS BENITO *is a specialist in the experimental testing of models and a leading figure at the Laboratorio Central de Madrid.*

"The problem of covering large areas using the minimum of intermediate supporting members is one which has appealed to architects and engineers for centuries . . ."

E. D. Mills

The sensation of grandeur, the admiration in a spectator produced by the comtemplation of a large covered space completely free of supports or intermediary columns has been sought by builders of all times. To this sensation there has been added the perfect visibility offered by such large spaces, always useful for meetings and, today, because of the freedom it offers in changing the floor plans of space distribution, for changes required in large factories by the continual advance of industrial technique.

Because builders did not have, until recently, materials with a high tensile stress—a problem solved with the arrival of steel and even better with reinforced concrete—and because they were familiar with, nevertheless, two structural forms, beams and arches, it was logical to think that they were compelled to use arches in the form of barrel vaults and domes as more suitable to the characteristics of the materials which they used. The Pantheon dome with its diameter of 142 feet 6 inches, the barrel vaults of many public buildings constructed by imperial Rome which have stood until our time, attest to this fact.

The necessity, perhaps faced by ancient technicians, of centering the thrust line as much as possible among the various sections of the shell in order that the materials work under compression led them to great thickness which, as the dead load increased, made the structural problem even more difficult. On the other hand, they did try to reduce the weight by countersinking ceramic jars in the mass.

These shells were usually supported at their periphery—on very thick continuous walls, required under good equilibrium conditions to absorb the inclined thrust, which is a necessary consequence of the curved forms. The intuition of the builders, as well as occasional disasters, must have taught them to oppose thrust between contiguous domes or vaults, or vaults and apses as in the Cathedral of Santa Sophia. This is a happy solution of this type of thrust transmission in which, with a dome 98 feet in diameter, and with arches and half domes skillfully interlaced, there has been built a structure of 98 x 220 feet without a single intermediate support.

The edges of groin vaults and cloistered vaults allow the thrust to be concentrated at isolated points. This was utilized by the masters of the Gothic style whose skillful handling of flying buttresses and abutments enabled them to produce a marvelous sensation of lightness in their cathedrals.

Much more could be said here on the influence of the materials in the structural forms and how they sometimes rebelled against man. We know that after St. Peter's Dome was completed, it was necessary to add various iron rings so that they might absorb tensile stresses in the lower parallels. We also know of the case of another dome which had to be built and rebuilt three times to attain a form, thickness and support which agreed with the materials used and the weights to be supported. Failures of which we have heard and some of which we probably do not know guided our ancestors in the conception of masterpieces and served to form an atmosphere favorable to the development of present-day theories.

The utilization of reinforced concrete in roofs of this type necessarily had to revolutionize the concept of them. It brought forth what the materials used up until that time had been lacking; a considerable tensile stress. It was no longer important that the sections of the monolithic roof work under simple or compound flexure; greater liberty was obtained in the forms and above all there was gained an advantage of prime magnitude: the thicknesses could be reduced. As the load to be supported by the roof was almost completely its own weight, the load diminished; this in turn made the working loads smaller, again allowing a reduction in the thickness and furthermore the weight of the roof.

This opened for the designer a path in the reduction of thicknesses which was checked in some cases only because of the risk of elastic instability of the structure, because of local buckling, or because, as the magnitude of the dead loads decreased, the relative magnitude of atmospheric effects increased (wind or snow loads and variations in temperature). Thus there has been developed the present type of thin shell of reinforced concrete in which the thickness, as for a sheet of paper, is very small compared to its other dimensions.

It is not surprising then that the first thin shells had the form of a dome or a cylindrical vault, isolated or linked. Thus with cylindrical shells of reinforced concrete there were covered, among others, the Planetarium of Jena (1924), the Market of Frankfurt-Am Main (1927), and the Frontón Recoletos of Madrid (1935). In some cases the shell was formed by a single cylindrical surface, while in others there were several cylinders which were joined together along one or another of their generatrices.

In some of the solutions the border generatrices served as continuous supports and in others the supports are made up by the two planes which contain the extreme directrices. In this way the shell, which in its origin has a close relationship to the arch, ended up with a working form similar to that of a beam.

The roofs of the markets of Algeciras (152 feet diameter), Basilea (170 feet), Leipzig (246 feet), and the one proposed for the Sport Palace of Stockholm (492 feet) are magnificent examples of spherical shells of reinforced concrete. In order to give an idea of the decrease in the dead load of the roof, it is noted that the dome of St. Peter's in Rome, (137 feet, 6 inches in diameter) weighs approximately 7 metric tons (7000kg), 1440 lbs per sq. ft., while that of the Market of Algeciras, somewhat larger in diameter, is not quite 50 lbs per sq. ft., that is 1/27 as great.

Cylindrical shells and domes appear to be the ideal forms to solve the problem of covering large surfaces with rectangular or circular floor plan. But when special conditions of illumination, support or form arise, caused many times by esthetic appreciation to be expressed in the shell, the creative architect, however little he gives free reign to his fantasy, will reach original solutions whose behavior will be quite different from the classic vaults and domes.

The roofs of the tribunes of the Hipódromo de la Zarzuela of Madrid, of the Brynmawr Rubber Factory of Wales, of the auditoriums of The Massachusetts Institute of Technology or of Karlsruhe, of the St. Louis Municipal Airport or the one projected for the Tachira Club of Caracas are some few samples of thin shells in which there has been found the solution to functional and esthetic problems with no fear of using the new forms of double curvature.

This overall review would not be complete if we did not cite a new group of shell structures formed by triangular folded plates. To this type there belong, among others, the roof of the Crystal Pavilion of the National Sporting Center of London and the structure of the chapel of the U. S. Air Force Academy.

The designer of a structure or a roof cannot be limited by calculation methods, but must use them to fix forms and dimensions.

Technicians well know the difficulty presented by the utilization of a general method which may serve to design thin shells of any form. In addition it is well known that the theories or methods of calculation currently in use are solely applicable to certain specific forms such as cylinder, paraboloid, conoid, shell, etc., which have been sanctioned by experience for certain pre-determined dimensions in plan. Within these special cases some theories assume that the material of the shell is homogeneous and isotropic; this agrees only approximately with reality, since this type of roof is constructed in the majority of cases of reinforced concrete. In the above theories, it is necessary to develop the calculations in agreement with the hypothesis of elasticity and to admit new simplifications which consist, for example, in supposing Poisson's coefficient of the material equal to zero and in other cases that certain stresses are nil or equal.

With cylindrical shells after some edge conditions have been introduced, a differential equation of eighth degree is obtained which reflects the equilibrium period. In some cases this equation can be solved. Then the forces that act on the various sections of the shell are known and the shell may thus be dimensioned in accordance with the theories of reinforced concrete or pre-stressed concrete.

When the form of the shell is double curved, if the mebrane theory is not admitted, and it is not always admissible, the calculation is quite difficult, especially when it is not a form derived by rotation or when the edge conditions are complicated. It is then necessary to turn to new simplifications of theories, in which results are obtained which have litle correspondence with reality and are quite difficult to evaluate.

From the above there is deduced the necessity of solving two problems of true importance in the design of thin shells:

1. After certain hypotheses have been accepted as valid to develop a theory or method of calculation, it would be quite useful to know the behavior of one or more thin shells designed according to them in order to compare them with the theoretical results and thus to contrast the degree of precision of the method used.

2. When a shell is given dimensions by approximate procedures not sanctioned by experience, before it is constructed one must predict sufficiently accurately the effects of the loads it is to bear or support.

Sonic determination of behavior of completed structures can be a magnificent source of information to solve the first problem. In relation to the second, one must recognize that the results of the auscultation, although very interesting, are the proof of the inevitable—the building stands or falls, but they are not useful to the designer in predicting its behavior.

As will be proven throughout the present work, testing of small models of thin shells furnishes satisfactory results for the two problems as stated. There is offered to the technician the opportunity to study the behavior of a structure with characteristics analogous to that conceived by him, the results of which are completely similar to those that might be obtained from the real structure.

Tests with Reduced Models. In general a reduced model of a structure is a scaled reproduction of the form and dimensions of the structure constructed

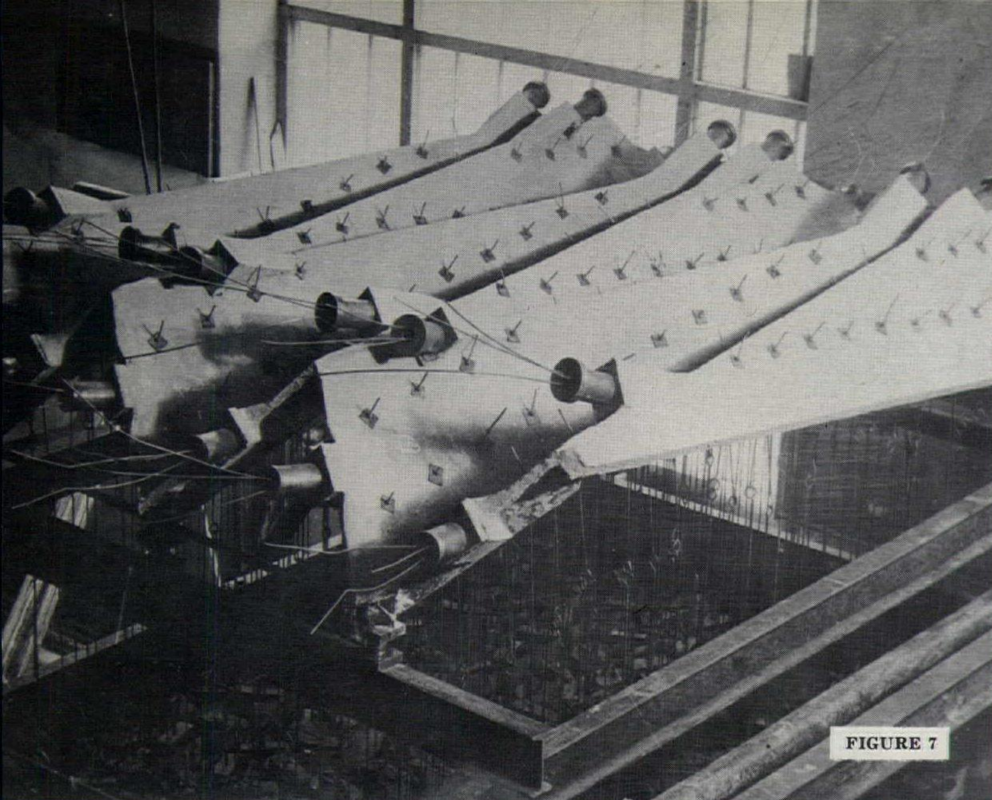


FIGURE 7

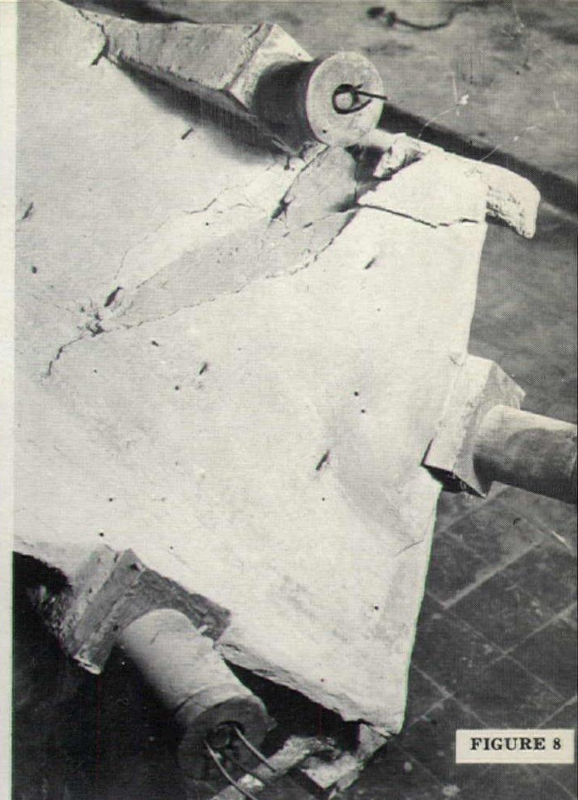


FIGURE 8

FIGURE 9

with appropriate material in order that when loaded suitably it will reproduce resistance phenomena and give results whose nature or magnitude was not known. Therefore, to carry out a study with a reduced model of a structure there must be available:

1. Adequate materials and methods which will allow fabrication of the model.
2. Procedures in order to subject the model to loads analogous to those which will act on the real structure.
3. Arrangements to measure with sufficient precision the displacements, rotations, or other magnitudes which may concern the real structure.

We see, therefore, the necessity of relating the experimental results obtained from the model with those of the projected structure for which it is necessary that there be a correlation, not only between dimensions, but also between loads and the characteristics of the material used in the model and real structure. In this manner, some experimental results will be able to be translated to the real structure from the reduced model which is being studied. This relationship between model and real structure is what is established by means of dimensional analysis.

It does not seem appropriate to introduce here the bases of dimensional analysis since they appear largely in books dedicated to this subject and the application of theorem 77 in the case of the reduced model of structures appears in The Student Publication, The School of Design, North Carolina State College, Raleigh, N. C., Vol. 6, No. 3.

From the aforementioned theorem there are drawn the following similarity relationships between the magnitudes of the model and the real structure. In these relationships the following values hold true:

σ	stress
E	modulus of elasticity
ϵ	deformation (strain)
p	external loads
c	displacement
l	length
F	concentrated loads
ν	Poisson's ratio

All these symbols are written with the subscript *M* or *R* indicating magnitude of the model or real structure.

With the exception of the last relationship which is equal in the Poisson's ratio of the model and the real structure, it is not possible to draw conclusions unless the problem is determined. But as eight different magnitudes are linked in five relationships, there is seen the necessity of giving three of these magnitudes as data of the problem.

At present, all the characteristics related to the real structure are known; therefore, we know the modulus of elasticity, the Poisson's ratio, and the relative density of the material, the dimensions of the structure, and the load characteristics either uniformly distributed or concentrated. We are then to fix the three magnitudes of the model in order that it be defined.

Normally in planning the model, its size is arbitrarily chosen, the material is selected (whose modulus of elasticity, E and Poisson's ratio ν are known) and the appropriate loading procedure. Thus we know the scale of length, modulus of elasticity and of forces with which the remaining scales of the previous relations can be deduced.

Therefore, in order that the similarity be defined, there must be chosen the size of the model, the material, and the loading procedure.

Selection of the Size of the Model.—In tests with reduced models, it is advisable that the latter be as small as possible within certain admissible limits since construction material is saved, necessary loads are smaller, they occupy less space and the time necessary for the experiment is generally shorter.

The causes which limit the degree of reduction of the model are twofold. On the one hand, at a certain size one is confronted with the impossibility of constructing given zones of the model. On the other, the reduction of the scale of lengths is accompanied simultaneously by a decrease in the relative deformations caused by the loads and consequently measurement operations become complicated if measurements are to be made with sufficient accuracy.

The Selection of Material to be Used in Construction of Model.—To produce a reduced model we can use various materials whose strength characteristics are quite different. Selection must be made while keeping in mind the purposes to be drawn from the test.

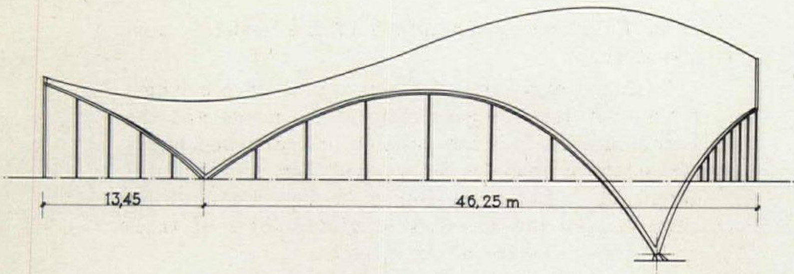


FIGURE 10

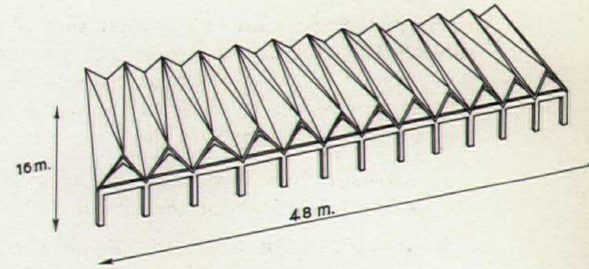


FIGURE 12

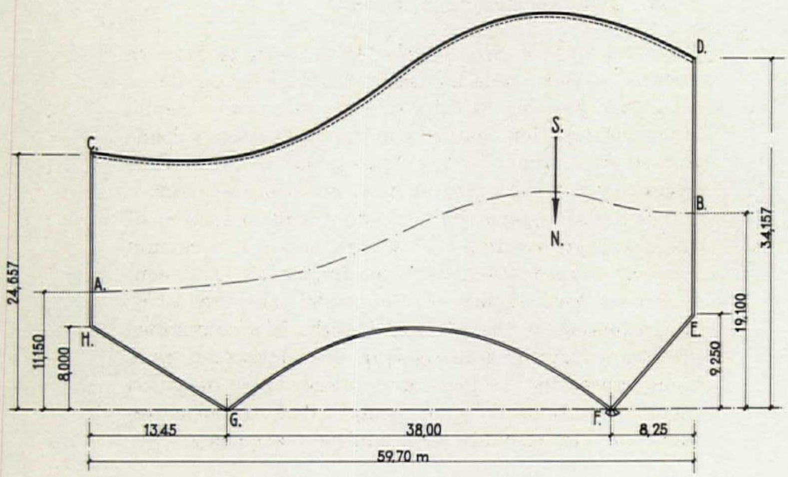


FIGURE 11

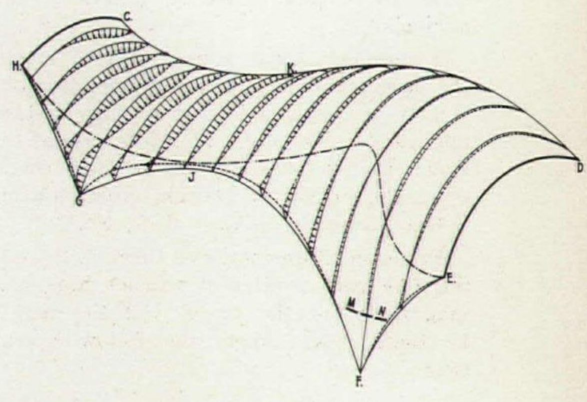


FIGURE 13

When we desire to predict the elastic behavior of the designed structure, it is indispensible that the material of the model and the real structure have identical characteristics. Therefore, if a reinforced concrete shell has been designed, the material of the model should be anisotropic and be composed of reinforcements and mortar whose moduli of elasticity are in the same proportion of those of steel and concrete. In the tests carried out by the staff of the Laboratorio Central de Madrid and which are described later, there are utilized in the construction of the model steel reinforcements and mortar of cement and sand. These produced a material with the same modulus of elasticity, Poisson's ratio and resistance to rupture in tension and compression as those which would probably be attained at equal ripening ages with the concrete of the real structure. In this way it is possible to study with a model not only the elastic or quasi-elastic behavior of the structure, but also its cracking and failure. This permits determining the safety factor to rupture of the real structure, a value which is most representative to express its operational value.

Construction of Model.—If it is desired that the model represent a structure of reinforced concrete when subjected to predetermined loads, and this throughout the elastic period as well as when cracking and failure through increased loads occurs, it is indispensible that the ratio between stresses and deformations caused in the model be completely analogous to those obtained in the real structure. If, in addition, there is desired the study of the phenomena of buckling which is of prime importance in shell structures and which are influenced largely by changes of form adopted by the structure because of the loads, it is deduced from dimensional analysis that the values of the moduli of the longitudinal elasticity and of Poisson's ratios of the materials used in the model and in the real structure must be equal.

The necessity of obtaining a material that fulfills all these conditions and in which the relative rigidities, in regard to thickness, are maintained in the same proportion that, in the real structure, produced the bonding of the concrete with the reinforcements, led to the study of the possibility of producing reduced models with a concrete whose maximum size aggregate was very small, in order to pour concrete in the very small thicknesses which the model would require. Following this line of reasoning we utilized for models a mortar composed of cement and sand suitably proportioned in order that the tension and compression rupture loading be equal to those of the concrete in the real structure for similar curing times. The physical composition of this material and an adequate proportioning assured that the moduli of elasticity and Poisson's ratios would have the same value in the model as in the real structure.

Within this mortar can be placed the reinforcing steels, produced to scale, which would have been provided with the real structure. These reinforcements can be replaced by small-diameter steel wires easily obtained on the market. These steel wires usually have an elastic limit and rupture load greater than those used for reinforcement in reinforced concrete. Nevertheless, recently we have been successful in overcoming this difficulty by heat treating said wires which decreased the values of those characteristics so that they were perceptibly equal to those of the reinforcement used in the real structure.

Although these reinforcements can be reproduced with sufficient precision, usually the same is not true for the cables provided in shells of pre-stressed concrete, since the great number of anchors, which usually cannot be reduced in scale, causes the number and distribution of the cables in the model to be changed as against the designed structure. In these cases the necessary variation (after consultation with the designer) usually consists of reducing the number of cables, which requires that the resultants of the stresses transmitted by the latter to the shell coincide in the model and in the actual structure.

After the material has been selected, the selection of the appropriate model scale is more simple. The thicknesses of the shells, which are small, are made even smaller in the model and it usually is this fact that limits the scale. It cannot be forgotten that in this thickness, the reinforcement sometimes is in two layers between which must be placed the cables for the post-stressing of the shell.

In making the model, a formwork is used, usually of plaster of Paris with a layer of shellac and slightly greased to keep the cement from adhering to it. On the formwork are placed a great number of nails whose points come out of the formwork to a length equal to the thickness of the shell. These nails fulfill a double purpose: they serve as a reference to assure that the thickness of the shell is that desired—we have succeeded in building models of different thickness (the thinnest of 6 mm) with error of some tenths of a millimeter (always less than a half millimeter). This has been found in the models after rupture when it is always possible to measure the thickness of the shells at different places. But, in addition, when the formwork is taken off, the nails which traverse the shell leave holes and there can be passed through them the wires from which hang the weights used to load the shells.

Loading Procedure.—When the model has been constructed there must be applied to it the loads with a distribution similar to that which the real structure would have, but at the same time with a rhythm of application also similar to that of reality.

The most practical procedure to apply loads distributed over a shell consists of hanging a great number of weights from it. This method, in addition to not making the location of the measuring apparatuses on the surface of the model as difficult as when loads are applied uniformly over the whole surface, has another great advantage since it allows us to observe the surfaces of the loaded shell, to detect the appearance of cracks and fissures, and to observe quite carefully the rupture process.

For applying loads in the laboratory the following procedure has been used for numerous years, always with good results:

The test model is placed on a tank two meters deep and somewhat larger than the model in width and length. Inside the tank are placed cylindrical receptacles 9 cm in diameter, 1.6 meters long and 9 kg. in weight which float when the tank is filled with water. These receptacles are suitably hung from the structure to be tested. As the tank is emptied very slowly, the structure is loaded progressively and uniformly over all of its surface.

By hanging these receptacles adequately spaced, there can be obtained various load distribution patterns. If they are placed very close together, up to 220 lbs. (100 kg.) per sq. ft. of loads may be attained and the shell can be loaded and unloaded as often as desired. If greater overloads are necessary, it is sufficient to ballast the receptacles appropriately. In the tests carried out at the Laboratorio Central, loads are reached slightly greater than 352 lbs./sq. ft. (1,700 kg. per m^2) by filling the receptacles with gravel. As is logical, on filling the general tank with water, the receptacles thus ballasted do not float and the shell is not completely unloaded; therefore, it is advisable to use this solution only when it is desired to reach the rupture point of the shell and to vary the live load between zero and 2,204.6 lb. (1 metric ton/m^2) for repeated tests.

This simple procedure requires only the auxiliary construction of a brick tank, the cost of which is very low, and a great number of cylindrical receptacles which will pay for themselves easily since they are used in successive tests.

Measuring Apparatus.—In a reduced model of a shell of reinforced or pre-stressed concrete there can easily be measured a displacement or a deflection with an error of less than .01 mm and revolutions with a precision of .0001 radians which is deemed adequate in practice. The measurement of deformations by means of mechanical or electrical strain gauges is also possible on the surfaces of the shells, although their usefulness is moot because the material is anisotropic.

When the shell is pre- or post-stressed, it is easy to anchor this type of reinforcement by means of visible dynamometers during the test which, therefore, allow the loads acting on them to be determined.

The magnitudes of revolutions, displacements, and deformations measured with increasing and decreasing loading, give a clear idea of the elastic behavior of the structure and the numerical values are of indubitable validity if compared with the values arising from the more or less simplified theories used by the designer. All this allows the formation of a criterion on the validity of the application of said theories to the case under study. But, in addition, if the loads are increased slowly until hair lines and open cracks appear, it is possible to unload the structure, to observe those cracks that close again and those that remain open, to interpret the phenomena, and finally to load the structure up to complete rupture. Total rupture may be very enlightening.

When these models have been constructed with a mortar with the same modulus of elasticity and the same rupture stresses as foreseen for the concrete of the real structures, the uniformly distributed loads per square meter in the latter must equal those of the model. Under the above conditions the uniformly distributed loads capable of producing rupture of the model will be equal to those which would cause the collapse of the real structure, so there can be determined experimentally its safety factor to rupture. It must be stated that in the rupture-load there will be included the weight of the hanging weights and that of the model which per surface unit keeps the same relationship with the real structure, just as it did for thicknesses.

Tests Carried Out.—As examples there are described below four studies of thin shells tested by the staff of the Laboratorio Central in reduced model.

To cover the central space of the Church of St. Felix and Regula in Zürich, the engineer, E. Schurbiger, with the advice of Professor Torroja, planned a reinforced concrete dome, elliptical in outline, post-stressed on the periphery.

The major axis of the ellipse was 24 meters, the lesser 7 meters, and the rise at the center 1.6 meters. Therefore, the ratio rise/major diameter was 1/15, clearly showing the slight curvature of the dome. In this type of structure the maximum admissible load is limited by the beginning of the buckling phenomena which must be produced in the direction of the lesser curvature. Because of the difficulty presented by rigorous calculation of the critical buckling load, the author of the project authorized the construction and testing of three domes as reduced models. The models were built on a scale of 1 to 10 and all of the reinforcements, as well as the post-tension rings which were joined to the shell by movable arrangements, were reproduced in the models. The graduated scale for the rings allowed post-tension forces to be varied. In Figure 1 can be seen one of the models during a loading test.

The deflections and revolutions in the dome and deformation in the rings were measured with symmetrical and asymmetrical loads. The force of post-stressing necessary to lift the crown of the dome from the formwork was calculated. Also studied were the effects which could be produced in reality by a small error in the leveling of the supports. Finally all three models were ruptured with a uniformly distributed load. Figure 2 represents the form of rupture of one of the models and in it can easily be seen the direction in which the buckling of the shell occurred. The load causing the buckling was determined quite accurately by the above described system.

In the second example, to cover the plant of a great Dutch factory, rectangular in floor plan, 80 x 264 meters, Professor Haas of the Technical School of Delft designed a shell of reinforced and post-stressed concrete. This shell, whose form can be observed in Figure 3, is composed of 22 equal elements, 12 x 80 meters, which are supported on three multiple rigid frames.

The shell of each element acts as a continuous beam of two spans of 40 meters with only two meters rise and 7 cm thickness made rigid with ribs every 4 meters.

The calculation of the safety coefficient to buckling used by the designer was applied for the first time to a shell of this type and of such dimensions. The designer wished to have the results of the calculation tested experimentally. For this purpose he authorized the Laboratorio Central de Madrid to construct and test to rupture a reduced model of one of the structural elements.

The model shown in Figure 4 was constructed to the scale 1/10, reproducing form and dimensions both in its exterior appearance and in its reinforcement (more than 7,500 reinforcements were placed in the small thickness of the shell). According to Professor Haas the necessary cables for post-stressing were replaced by high resistance steel wires.

During these tests increasing and decreasing loads were applied to the shell. Displacements and deflection at a great number of points in the structure were measured. The appearance of cracks was observed and finally rupture occurred. Figure 5 represents the model after rupture and Figure 6 a view of the structure under construction by Nedam Havenwerke.

In a nave 48 by 16 meters of the Universidad Laboral of Tarragona, Professor Torroja in collaboration with the engineers F. del Pozo and A. Páez developed the project proposed by the architect A. de la Vega for a spiral shell of reinforced prestressed concrete (form can be seen in Figure 12).

The shell is formed by a series of equal flat triangles 12 cm thick which are fixed elastically together and are supported on triangular frames

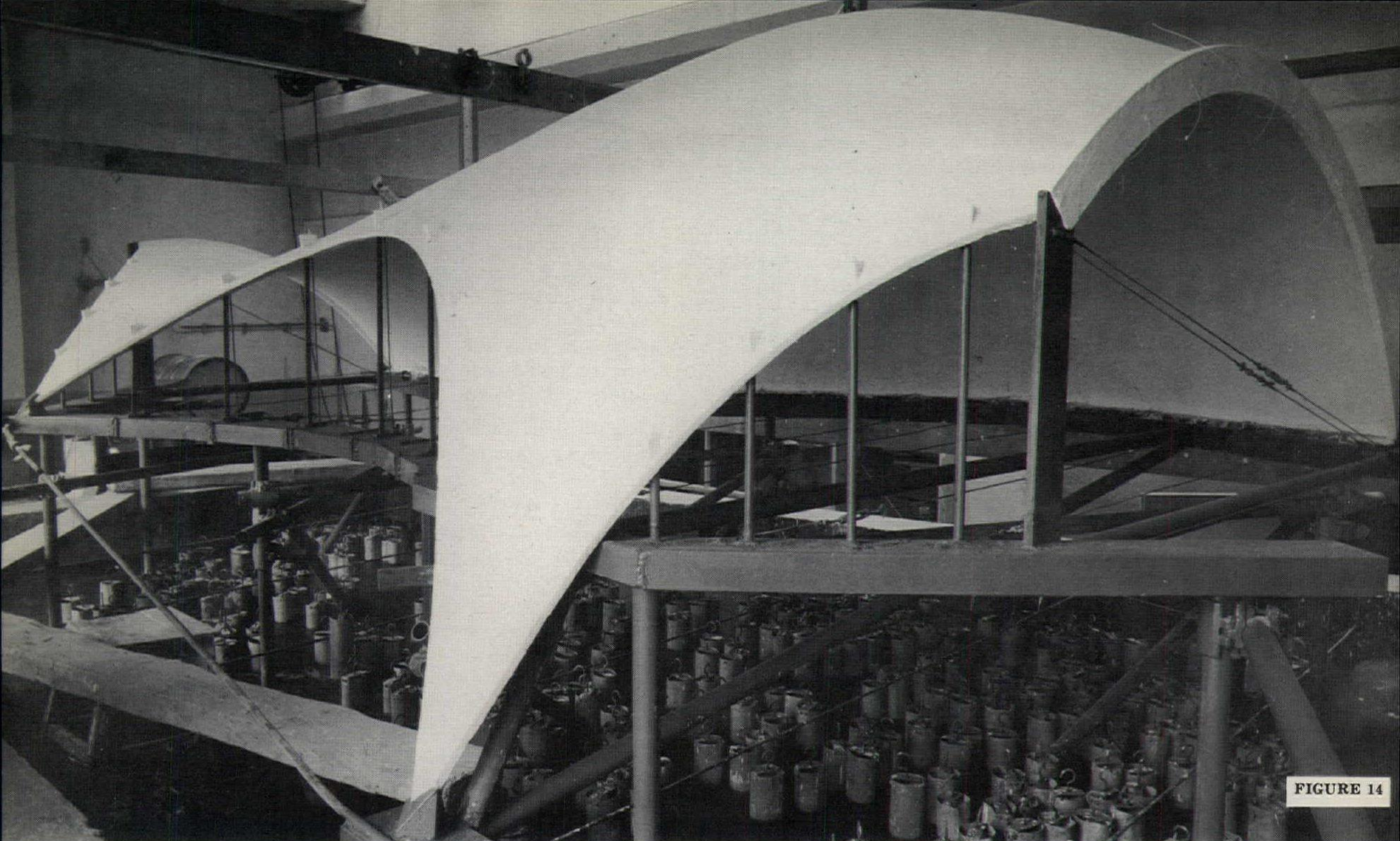


FIGURE 14

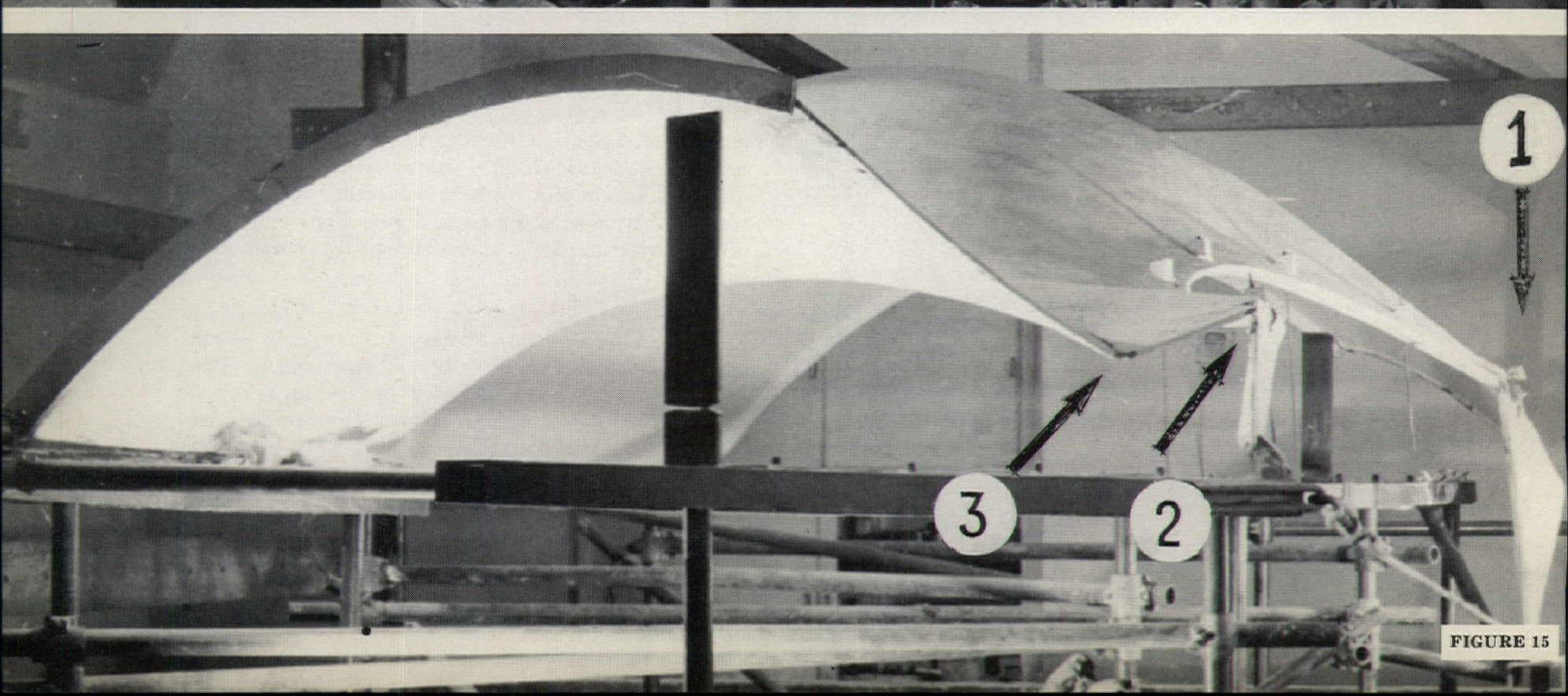


FIGURE 15

placed in the widest facades of the building, which was completely free of supports in its interior.

Repetition of elements of equal resistance allows reproduction in the model of a group of them so that the influences of the free periphery of the last triangles of each end do not reach the central part. The model in Figure 7 was constructed to the scale 1/15 and as in previous examples, reinforcements designed by the author were reproduced in it. It was subjected to the effects of uniform increasing and decreasing loads, then the loading was increased to the formation of cracks and finally to rupture. Figure 8 gives a partial view of the rupture. Figure 9 shows the structure in an advanced stage of construction. For the anchoring of the post-stressing cables, both in the model and the real structure, Barredo anchors were used.

Among the various installations and constructions provided for the Tachira Sporting Club of Venezuela is a shell of reinforced concrete designed for the social room by the Venezuelan architect, Señor Fruto Vivas. The complicated structural shape, practically unapproachable by rigorous analytical methods, induced the designer to ask the technical help of Professor E. Torroja as a specialist in the calculation of shell structures. The methods utilized by Professor Torroja to obtain the dimensions of such a complex structure required testing a reduced model.

On a terrain with very marked level changes there were projected sporting fields, swimming pools, halls, gardens, etc. The social building included a large ballroom, meeting rooms, and rooms for the principle services of the club, leaving the remainder to be placed in another building. The largest parts of that building would be the large reception room and another more intimate room partially separated from the first, although connected to it on the outside, especially on the north side, a direction from which there could be enjoyed beautiful vistas and countryside. When the architect conceived both units on the same scale he utilized the distinct necessity of height felt in them to mark the separation between the two units. There was also foreseen the construction of some floors or mezzanines completely independent of the shell; therefore, the design of the latter should be treated with complete independence of the rest.

The form, adopted in principle by the architect, evolved through successive changes of ideas with the specialist and as the fruit of his first collaboration, indicated in the projection and ground plan in Figures 10 and 11 was adopted. This surface was defined by AB for which there was known its space coordinates and which was the geometrical locus of the vertices of the catenaries belonging to the same equation, all situated in vertical planes parallel to the N-S direction in Figure 11. The ground plan of the periphery was sufficient to define the latter in space.

Peripheries DE and CH were made rigid by means of very light metallic structures. The border GF was reinforced by increasing the shell thickness and this produced a distorted arch. CD probably will rest on the ground and sides GH and FE on supports.

The necessity to study the behavior of the structure under loads convinced the designers that only experimentation, very ample and detailed on a reduced model of the thus complicated structure could confirm or advise modification of the adopted forms and dimensions. Consequently, the Laboratorio Central de Madrid was charged with the accomplishment of this experimental analysis. The results of this experiment are summarized below.

The project designers had calculated that between the shell cover, waterproofing and insulation, the dead load would be about 53.7 lb. per ft.^{2} (240 kg. per m^2) and the safety coefficient of rupture would be equal to or greater than 2.5. Besides, the wind effect would be taken into consideration as live load, vertical and equal to half of the dead load added to the latter and occupying half of the shell along the N-S line which would have to be carried without cracking, in any part of the structure.

The test began with the application to all the surface of the model of the shell, Figure 14, of a uniform load of 49.6 lbs. per sq. ft. (240 kg/m^2). After having observed in the fleximeters, placed appropriately, that deformations had ceased, and that, therefore, equilibrium had been reached, the overload was increased up to 74 lbs. per sq. ft. (360 kg/m^2) and the deflections were measured at 40 points on the shell and on the edge.

After the shell was unloaded, its elastic behavior was proved as no permanent deformations were observed. It was later loaded and unloaded two more times up to 74 lbs. per sq. ft. and the measurements repeated. Later the measuring instruments were removed and the loading and unloading procedures repeated until deflections were obtained in 120 more points. Finally, the uniform load was increased up to 600 km^2 (125 lbs. per sq. ft.), with satisfactory results.

When the uniform loading tests were finished, the shell was loaded asymmetrically in order to reproduce the effect of its own weight and the wind thrust. No cracking of the shell could be observed in any of the successive tests. Figure 13 shows the above results in a perspective of the shell before and after deformation.

After the following results were obtained, the shell could have been overloaded to rupture point. Instead of this, and as the safety coefficient obtained was acceptable, it was preferred to study the possibility of supressing the bracing designated MN in Figure 13 which prevented the beginning of buckling in this practically flat zone. After this bracing had been removed, there was applied to the shell an increasing uniform load per surface unit. At 114 lb. per ft.^{2} (510 kg/m^2) overload, the zone from which the bracing had been removed buckled and the rupture of this element spread throughout the whole edge of the shell in the north side. Because of the loading system used, the great deflections caused during the collapse caused the loading to cease automatically; therefore, it was possible to take a photograph, Figure 15, after removing the wires which served to apply the loads and some of the tension members. In this figure Arrow 1 indicates the zone where rupture began.

When the continuity of the central rigidity arch was interrupted Zone 2 failed almost simultaneously and, lastly, Edge 3.

As a consequence of all these tests, there was accepted as the definitive form of the shell the form projected with transverse sections in catenary, reinforcing arches at the east and west sides and the central part of the north side, bracing to avoid buckling in the zone nearest the lowest support of its shell, and the edges of the north facade braced and post-stressed.

Although we could give other examples, we hope that the above will serve to demonstrate the advantages of experimentation on reduced models up to rupture when attempting to design shell structures.

CONCLUSIONS

Throughout this paper there have been shown the advantages offered by experimentation on reduced models of thin shells. Below they are summarized.

With reinforced concrete models, it is possible to study the behavior of the structure in the elastic period, in cracking and in rupture. There can also be determined the safety factor to rupture of the designed structure. The examples given above are a good proof of this.

In addition to the important advantages cited above there is the possibility of observing the structural behavior of the structures designed and their method of reacting under loading effects. The thorough understanding by the designer of his work is, in our opinion, the most interesting result of the model test. From them would arrive at times indications for the reshaping of certain danger zones, perhaps from inadequate dimensions; and also a confirmation of the skill of the designer. In any case, the results are equally satisfactory because they give the necessary proof for any well-designed structure.

ACKNOWLEDGEMENTS

The author wishes to thank the personnel of the Laboratorio Central de Madrid and very especially its Director and his former teacher, Professor Eduardo Torroja, of the Assistance in Public Works, D. Antonio Sánchez Vegas and Don Fernando Rondriguez Merodio, charged with the construction of the concrete models, and the industrial expert, D. Anselmo Moreno Castillo, charged with the measurement of all the models constructed, for all the favors and guidance given him. To them belong the principle merits of this work.

PRESTRESSED SHELLS

I. M. HAAS *has taught at the Technische Hochschule in Delft for a number of years and done important work in the development of prestressed thin shells. He has written widely on the subject.*

The title refers to shells made of prestressed concrete. It means applying the advantages and possibilities of prestressed concrete to shell construction.

In shell design the introduction of the prestressing technique has indeed led to several important advantages. First of all it enables the use of larger spans and thus the design of structures covering large areas unobstructed by columns is possible.

Secondly it has a favourable influence in the delay of cracking. Fissuration due to shrinkage, unforseen cracks due to creep, temperature changes, etc., may be delayed or avoided by applying a few prestressed cables to a reinforced concrete shell roof. A partial prestressed construction will thus be obtained. In climates with large relative moisture content and where watertightness is of primary importance we find this application of useful value. In the Netherlands northlight shells of only 66 ft. span were prestressed, partly for this reason.

Thirdly it provides us with an adequate and perhaps powerful means to stabilize or fortify the form or shape of a shell. Mostly shells when loaded to rupture will give away because the curvature is diminishing or edges are sagging and giving away. Application of prestress may greatly prevent or at least retard these phenomena and in this way add to the rupture-strength of the structure. Retaining of shape in general means that buckling takes place at a far later stage in the process of loading. These considerations especially apply when diverse or awkward forms are created and these should be designed.

Prestressing may also greatly aid to prevent sagging lines which may spoil the appearance of the boundary lines of a shell structure.

Furthermore it provides the designer with means to facilitate the assembly of precast elements, that is of pieces which may be assembled to make a whole. The completed cover may then behave like a shell. (1) *

It has almost become commonplace to state

*Numbers in parentheses refer to references listed on p. 25.

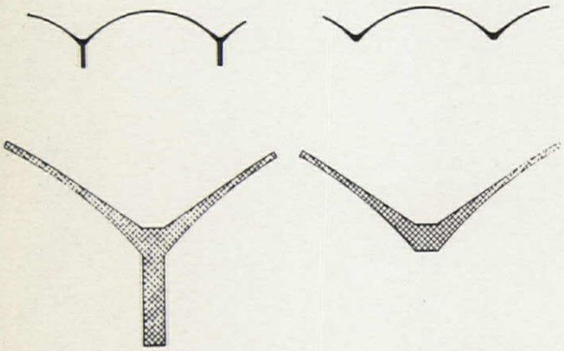


FIGURE 1

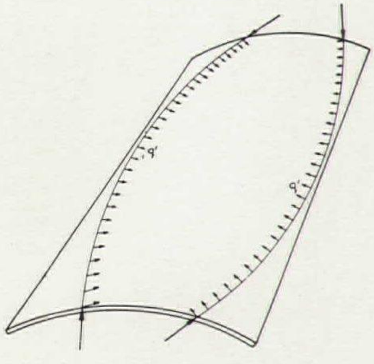


FIGURE 2

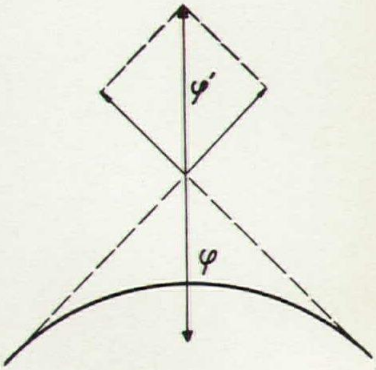


FIGURE 3

that in prestressed concrete tensile stresses are eliminated by the introduction of prestress.

A broader definition calls for: "a concrete in which there have been introduced internal stresses of such magnitude and distribution that the stresses resulting from given external loadings are counteracted to a desired degree" (definition of the A.C.I.).

From a standpoint of stress analysis a compressive stress has been introduced in advance. This stress will remain, although its magnitude may diminish somewhat due to shrinkage, creep, etc.

It is obvious that in the beginning prestress was applied to those parts of the structure where the tensile forces were prominent. Shells used in industrial buildings are usually of the barrel-roof, north-light (shed), or butterfly type and are often equipped with edge beams. These beams will often have a high percentage of reinforcement. Consequently these members will be the part of the structure where honey-combing occurs and cracking may be observed. Thus the first applications of prestressing went to the edge-beams. Several shells in which these beams have been prestressed have been built, (2). To mention some early examples: a barrel-roof spanning 131 ft. made at Karachi-Airport; at Bournemouth (Great Britain) hangars with 150 ft. span; in the Midlands (Great Britain) north-lights of 100 ft. span.

In all these cases prestressing was done using the post-tensioned method. If it is applied after the whole shell structure has been poured, the longitudional deformation (shortening) of the edge beam will effect the shell proper. As a result large shearing stresses will occur at the intersection of beam and shell. These stresses may be provided for by using mild steel reinforcement.

A step forward was made when the whole shell body was involved in the prestressing technique. It was Guyon (3) who called attention to the use of curved cables in this relation. Bent-up cables were used, their effect being two-fold. First, the deadweight and permanent load could thus be carried directly by the cables.
Second, in introducing a prestress directly in the shell body the shearing forces will be diminished.

In order to prestress the whole shell body a change of the cross-section was needed. In the case of a barrel-roof the edge-beam underneath had to be omitted. Only a local thickening near the edge was needed. Looking back to the old fashioned shell with edge-beam-construction it may be seen as an evolution into a better type. The disturbing edges had to be abandoned because a curved cable cannot follow the abrupt change at the intersection of shell and beam (fig. 1).

If such a bent-up cable is laid in a curved shell the result is a cable which theoretically has a double curvature. This means more friction-losses and un-

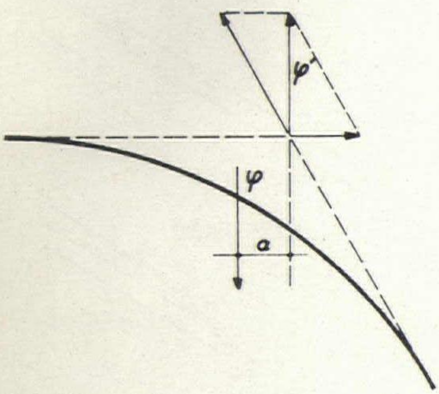


FIGURE 4

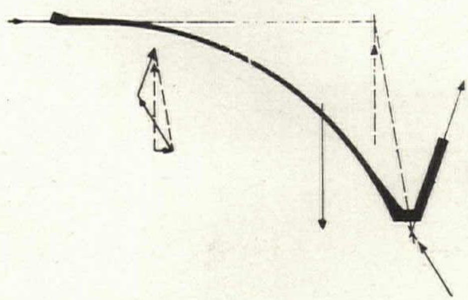


FIGURE 5

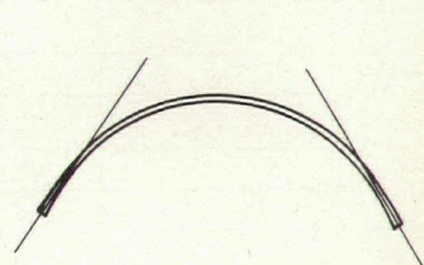


FIGURE 6

desirable prestress components on the shell-body (downward forces and tendency to flatten the shell). For these reasons the shell thickness is often gradually increased towards the edges in order to have less curved regions near the longitudinal edges.

It is a property of the curved cable that it will exert radial forces to the concrete surrounding it. In the case of a cable being curved in the plane of the shell the radial forces (q^1) in this plane may act to counter-act the component (q) of dead weight and permanent live load (fig. 2). Arrangements may be such as to fully neutralize these components: then $q^1 = q$. By approximation the small longitudinal components of the radial forces q^1 are neglected. If $q > q^1$ the influence of dead weight etc. is minimized to an extent proportional to $(q - q^1)$. All stress functions (bending moments, shear forces) will thus be reduced in proportion to

$$\frac{q - q^1}{q}$$

If the shell is taken as a whole, it may be considered as hanging on the cables. The cables may be so arranged that $Q = Q^1$ (fig. 3). In case an edge beam would have been provided, this beam probably would not have to carry any surcharge as it may be taken directly by the prestressed shell itself.

It of course depends on the ratio of stiffness of the shell and of the beams.

Equilibrium between the upward forces and the dead weight etc. may also be established for non-symmetrical cross-sections, although a transverse bending moment will remain, its value being $M_Q = Qa$ (fig. 4). When north-lights are fully prestressed, namely, including the gutterbeam, there will be three instead of two groups of cables, viz., in the upper part, in the lower part and in the free flange of the gutterbeam. By arrangement and also by choosing the number of cables in each group the vertical sum of the components may be considerably influenced. It means that the value of the transverse bending moment ($M_Q = Qa$) is influenced as well (fig. 5). In this manner an ingenious way to control the transverse bending moment is found. These tricky M_Q-moments are a source of uncertainty and speculation. Large differences found when numerical values and zero-points occurring in the moment-curve are calculated according to various theories and compared. These variations may be explained by the approximations or simplifications introduced in these theories of shell computation.

As long as these values remain comparatively small, little attention has been given to them. For

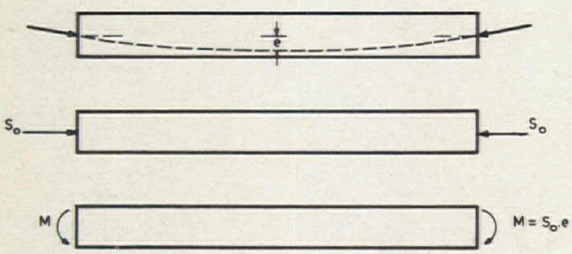


FIGURE 7

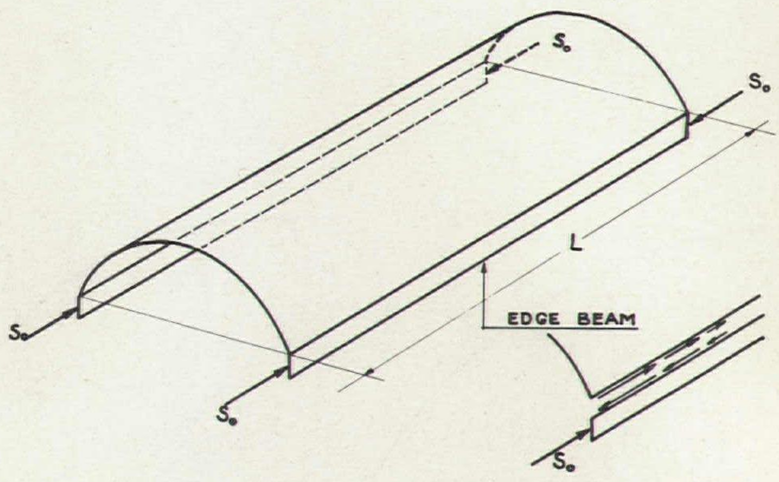


FIGURE 8

shells of long span they become larger and troublesome.

If the design may be arranged so that q^1 almost equals q and therefore $(q - q^1)$ represents a small amount, these difficulties are greatly reduced and become almost negligible. As stated earlier there will remain a bending moment directly derived from the prestressing forces and therefore well known and easily controlled. The identity $q^1 = q$ actually holds true for the regions of the shell in which the cables are located. As a matter of fact it is assumed that these parts are noncurved (fig. 6). Therefore, it should not be expected that for the region of the shell situated between two groups of cables $M\phi$ will be nil if for a symmetrical section $q^1 = q$.

Although these deviations may have qualitative significance, as to quantitative values little difference has been found. It may be of interest to mention the construction of a north-light shell of 2 x 131 ft. span and 39 ft. width, recently constructed and designed in prestressed concrete in accordance with the principles here exposed. The transverse bending moment could be taken care of in such a way that the 2¾ in. thick shell was reinforced with a single wire mesh of 8 x 8 in. gauge, of soft steel of 0.3 in. thickness. (4).

Application of prestress means that large compressive forces are added. This may lead to buckling at an earlier stage than expected.

In this connection the influence of creep should be studied, especially in its effect on the deviation from the longitudinal axis and also on account of the flattening effect on the cross-section. A preliminary study and some research have been done in this respect (5).

The application of prestress on a structure means the introduction of an external set of forces in addition to the live load etc. The way in which these outer forces are introduced in the calculation depends on the type of prestressing. If merely the edge beam is prestressed it may be sufficient to add to the boundary conditions a shearing force operating along the edge of the shell (fig. 7).

In numerical computations the load is generally developed in Fourier series. This may be done with comparative ease. For dead weight and uniformly distributed live load it will in general be sufficient

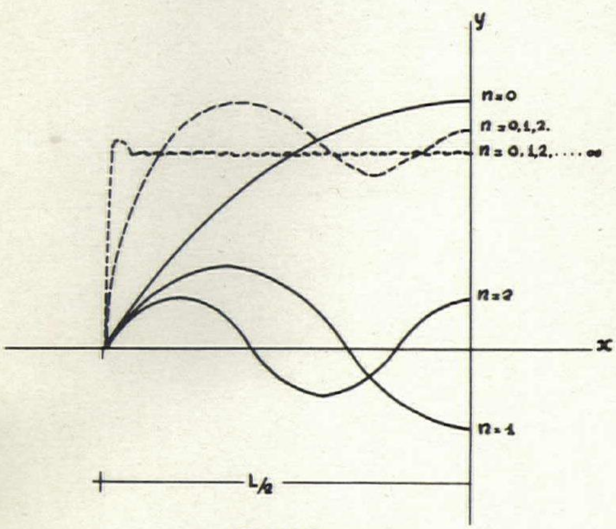


FIGURE 9

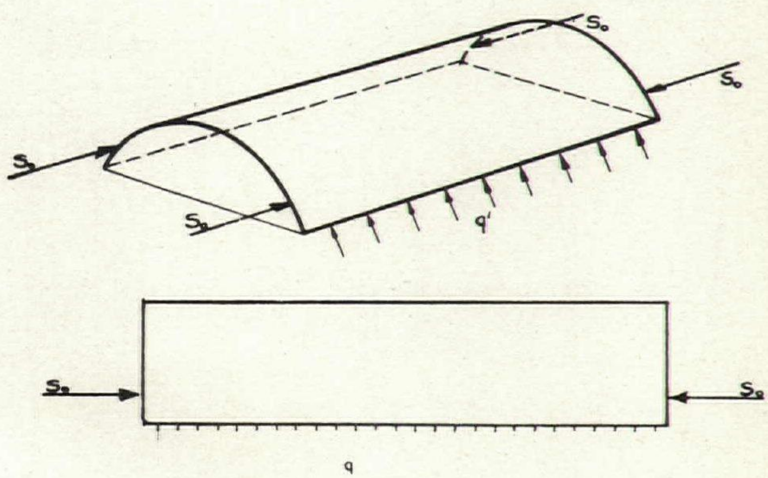


FIGURE 10

to restrict the Fourier analysis to the first term of the series.

In the case of the edge-beams being the only members prestressed, the resultant of the prestress should be replaced by a force centrally applied and by a bending moment (fig. 8). The linear force (prestress S_o) may then be expressed in a Fourier-series of the type:

$$S = S_o \sum_{n=o}^{n=} (-1)^n \frac{\cos a_n x}{2n+1}, \text{ in which } a_n = \frac{(2n+1)\pi}{L}$$

The stress function for the uniform compressive stress will be:

$$\sigma = \frac{4S_o}{F} (-1)^n \frac{\cos a_n x}{2n + 1}$$

where F is the cross section of the beam.

For n = 0 we get the first term of the series:

$$\sigma_x = \sigma_m \cos \frac{x}{L}$$

At the middle section (x = 0), from the equilibrium between prestress and the compressive stresses, it follows that:

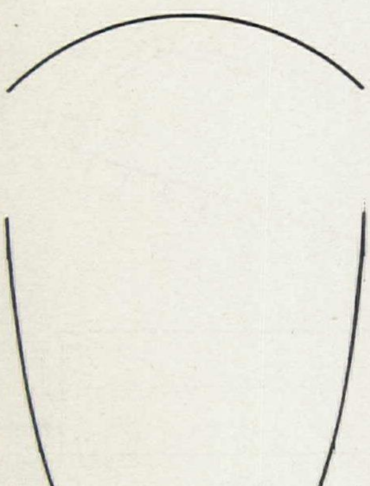
$$\sigma = \frac{4S_o}{\pi F}$$

There will be some agreement for the other cross sections and none at all for the end sections. For $x = \frac{L}{2}$ the stress will be zero against a full prestress S_o.

If more terms (the 3rd and the 5th harmonic at least) are taken into account, the agreement will be satsifactory except for sections near the end (fig. 9). This may sound unsatisfactory from a mathematical point of view but it is acceptable in this case. Near the end sections there is a large dispersion of stresses due to the concentration of forces at the end-anchorages. The contradictions may also be solved by restricting the problem to the range:

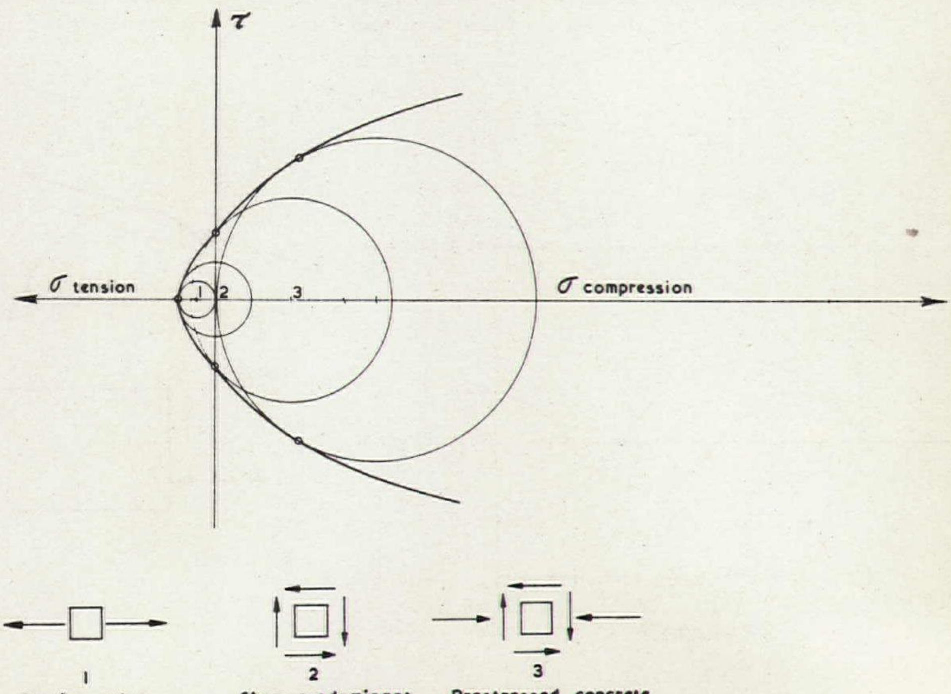
$x = -0.45$ L to $+ 0.45$ L

Schmausser follows another course (6). Though



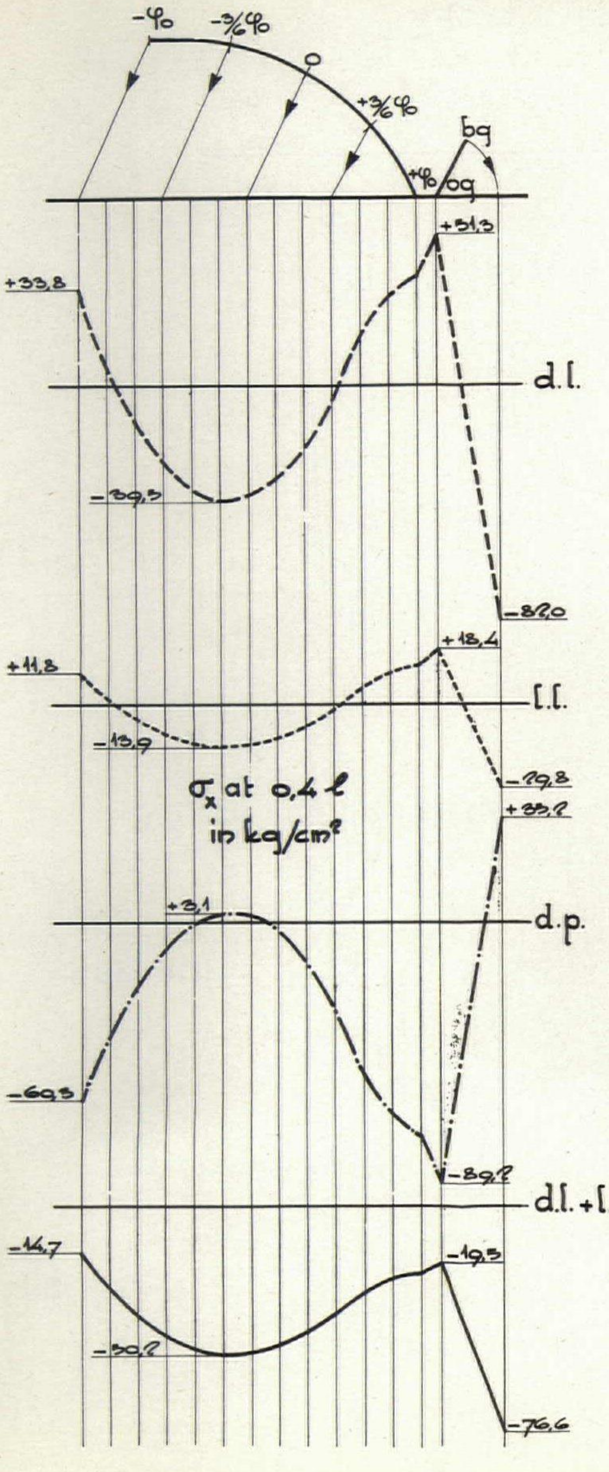
σ_x stresses at middle section due to prestress S_o

FIGURE 11

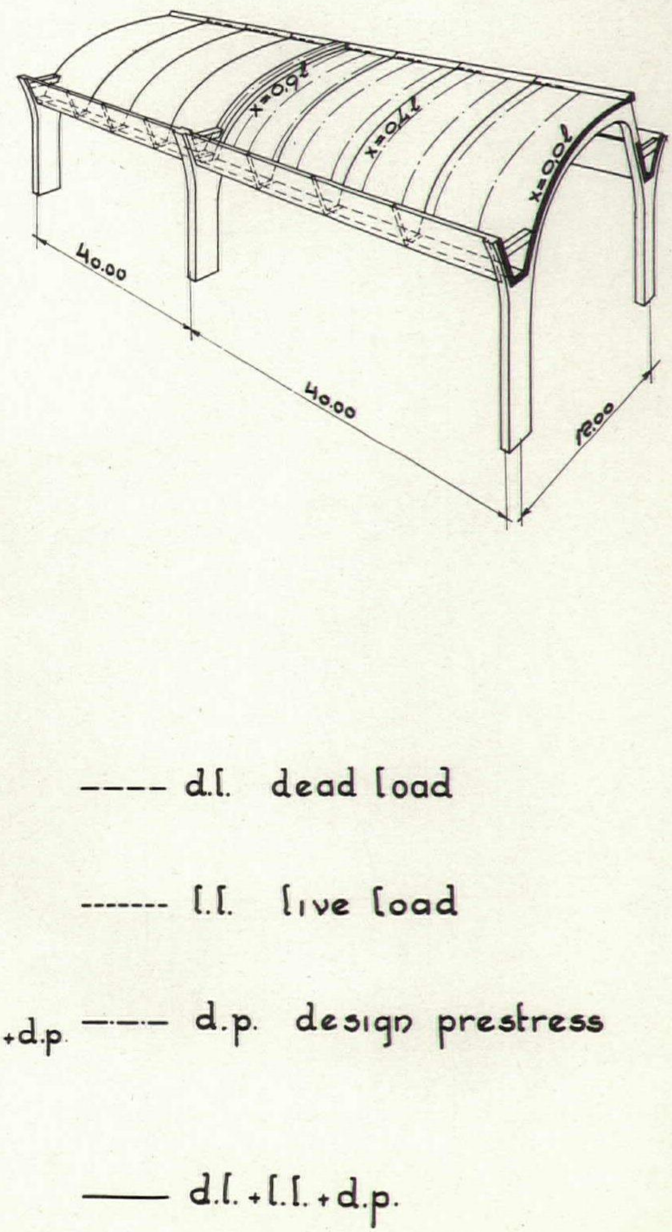


Ultimate stress diagram for concrete

FIGURE 12

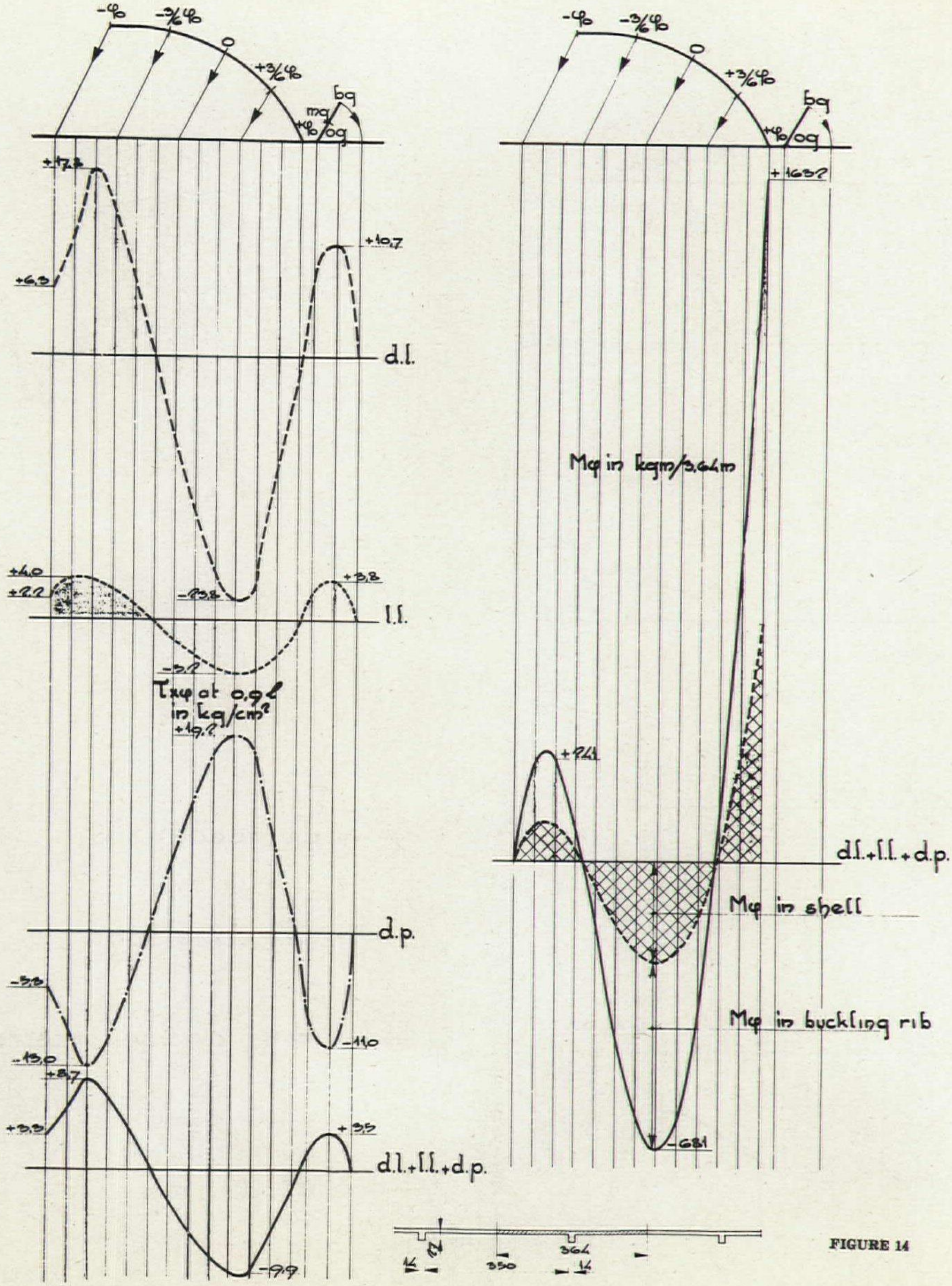


-φ₀
-3/6 φ₀
0
+3/6 φ₀
+φ₀
bq
aq
+51,3
+33,8
d.l.
-39,5
-82,0
+18,4
+11,8
l.l.
-13,9
-29,8
σx at 0,4 l
in kg/cm²
+33,2
+3,1
d.p.
-69,3
-89,2
d.l.+l.l.+d.p.
-14,7
-19,5
-30,2
-76,6



x=0,9l
x=0,4l
x=0,0l
40.00
40.00
12.00
d.l. dead load
l.l. live load
d.p. design prestress
d.l.+l.l.+d.p.

FIGURE 13



d.l.
l.l.
Tx φ at 0,9 l
in kg/cm²
d.p.
d.l.+l.l.+d.p.
Mφ in kgm/3.64m
Mφ in shell
Mφ in buckling rib
d.l.+l.l.+d.p.

FIGURE 14

the problem may be solved with the help of a Fourier-analysis—as is admitted by him—one is restricted to definite boundary conditions. This refers to the end-sections of the shell and may denote that shells extending over many bays (like continuous beams) will be difficult to solve following this scheme. Therefore, he proposes to replace the shell by a prismatic body (like a hipped plate structure). These structures composed of a number of plates subjected to lateral loads in their plane may be analyzed for any form. The amount of computation work to be done increases rapidly with the number of breaks, that is with the number of ribs chosen. In order to get results which come sufficiently near to the exact values this number should not be too small; especially when lateral forces are involved the number should be larger than when a comparison is made for dead-load and uniform perpendicular loading in general.

Little agreement will be found when the transverse bending moments are plotted for the true shell and for the hipped plate structure, which replaces it. This might have been expected when the fluid course of the $M\phi$ moment in a shell is compared with the type of bending moments encountered in continuous beams. By applying the theory of probability these large discrepancies may be adjusted.

From an example given in his article the agreement is satisfactory. Nevertheless it does not seem necessary to step back to the prismatic structures as there are other methods to master the difficulties. These have already been applied in several cases for shells prestressed and built continuously.

Returning to the subject of fully prestressed shells — using curved cables in the body of the shell —it is found sufficiently accurate to replace the cables by a scheme of two sets of forces, viz., linear forces S_o on the end-sections and a set of uniform loads of $-q^1$ (fig. 10). For the prestress S_o will be chosen such a value that at a critical section the

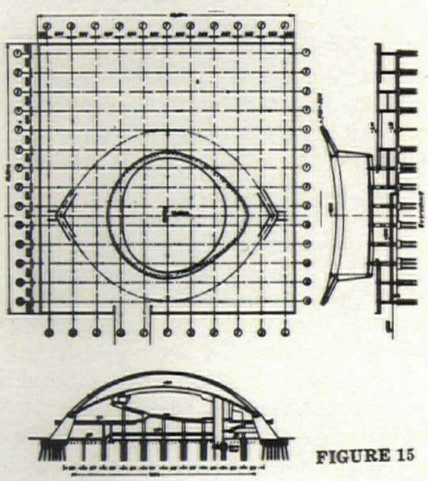
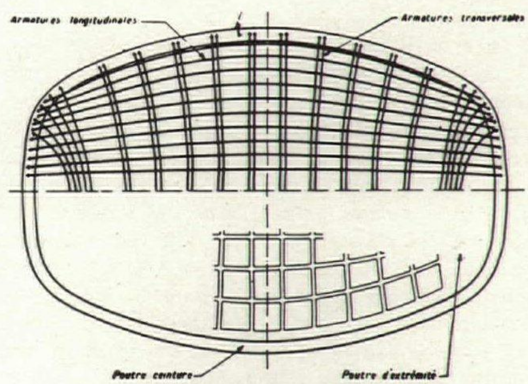


FIGURE 15



Arrangement of Prestressing Bars.

FIGURE 17

FIGURE 16

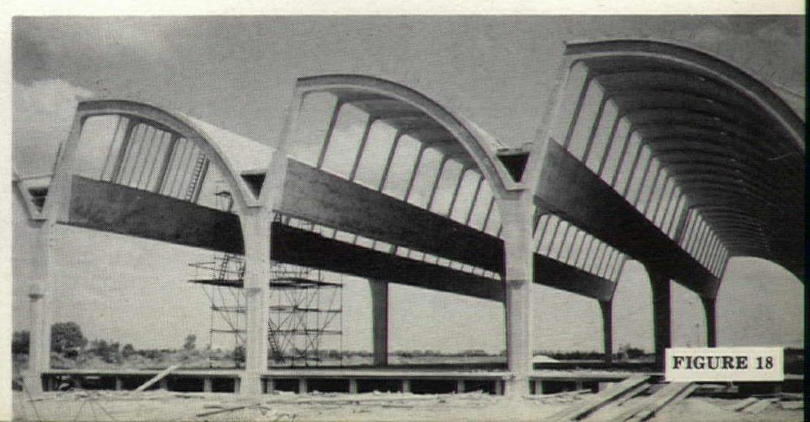


FIGURE 18

stress σ_x at the edge will be equal and opposite to the stress σ_x created by the dead load given live load; the large stress at the edge will thus be neutralized to zero; the stresses at and near the top will be less affected (fig. 11).

In general the ultimate tensile stress created by the design load at the edge should be neutralized by the compressive stress due to the prestress. This should be done for the prestress minus the relaxation losses; for simply supported shells the middle section should be considered.

In the case of non-symmetrical sections the problem is more complicated and several solutions will be possible.

Before giving an example of an executed prestressed north-light, attention is called to one important fact unsufficiently worked out in the beginning of this paper.

In concrete structures rupture mostly occurs through bending or shear. Failure through bending moment has sufficiently been investigated and formulae for the bending moment at rupture have been developed.

If shear is predominant, for instance, near a support, the rupture hypothesis as formulated by Mohr is mostly followed.

According to this theory failure is dependent on the shearing stress and the normal stress simultaneously occurring at a point. For various combinations of a normal and a shearing stress Mohr's circles may be constructed. (8). For these combinations (tension, shear, etc.) critical stages may be found for which rupture due to axial or angular deformation may take place. To these an enveloping curve may be drawn upon which the ultimate rupture-conditions will be found. For a combination of a normal and a shearing stress the point which this locus (fig. 12) has in common with the particular Mohr's circle represents the critical condition of rupture.

For reinforced concrete the circle indicated by 2 represents the shearing conditions in the neutral axis Then $\tau crit = \epsilon tension$ will be the ultimate rupture strength. For prestressed concrete the circle indicated by 3 is representative for its behaviour. Together with a shearing stress there will be a compressive normal stress. From geometrical deduction it will be found that $\tau crit = 0,5 \sqrt{\epsilon compr\ \epsilon tens}$ in which the ϵ's are principal stresses. From tests it appears that 0, 6 is a more correct coefficient. If in case of rupture we assume $\epsilon compr — 12\ \epsilon tension$ we find that $\tau crit — 2\ \epsilon tension$.

From this analysis it is clear that the resistance against shear of prestressed concrete (sufficiently prestressed) amounts to twice that of reinforced concrete. This conclusion has a special bearing on shell structures. For shells the shearing stresses are critical at an earlier stage than for beams; in a design they often govern the dimensions. Therefore, prstressing is of particular importance for shell structures and may greatly increase their resistance at rupture.

For a north-light shell continuous over 2 x 131 ft. the stresses are the result of dead load, live load (surcharge) and prestress. If the moment curves for dead load and uniform surcharge are compared there must be congruity as to shape. Conformity may also be noted if the diagrams for dead load and prestress are compared, (fig. 13). This may readily be understood if it is remembered that in the calculation of the stresses due to prestress a uniform load formerly called q^1 has been introduced. It should be kept in mind that the sign of this stress should be reversed. Furthermore it should be understood that this stress is composed of two elements: one due to the q^1 loading and one due to S_o compression. In this case the latter part is comparatively small.

The superposition of all stresses results in a compressive stress over the entire section. Another diagram (fig. 14) shows the distribution of t_{xy} at a section near the support (at $x = 0.9$ L) plotted for the contribution of dead load, surcharge, prestress, and a final summation. Also shown is the diagram of $M\phi$ (transverse bending moment) and its distribution between the shell proper and the buckling (stiffening) rib. Pictures of the completed structure are added (fig. 15, 16).

Prestressing has not been confined to single curved shell roofs. The same principles may be applied to double curved shells as well. As a matter of fact a few have already been constructed along these lines and several projects are in the process of being erected.

An example of an unusual prestressed roof is found in an exhibition hall at Karlsruhe (Germany). Covering an area of 3800 sq. yard the roof represents on plan an ellipse with a major axis of 240 ft. and a minor axis of 160 ft. Around the periphery is a tie-beam supported on 36 columns of varying height so that the upper surface of the beam follows the shape of a cylinder concave upwards. The columns under the beam are equally spaced at about 18 ft centres. The roof has the general shape of a saddle; and has the appearance of a part of a hyperbolic parabola. Along the major axis of the ellipse it has the form of a catenary rising 14′ 6″ from the centre to the edges but transversely (across the minor axis) it represents a very flat arch with a rise of 4 ft. The slab thickness is 2¼″ prestressed in two directions by 1″ diameter high tensile bars arranged as shown in fig. 17

There appeared to be a large amount of prestress in both directions which may be explained from the small rises in the curved surfaces. By increasing these a much more economical structure would be the result.

Sufficient pitches in both directions of the double curved surface have been provided in the shell roof of the Berlin Congress Hall (9). The geometric shape of the structure resembles a parabola rotating around an axis. The parabola is fixed in space by the two springings and the centre of the roof. The shell is limited by two arches. The plane of each makes an angle of 28 degree with the horizontal. The two inclined arches cross each other at two springings where they are supported. For stability reasons this is unsufficient. Therefore a solid curved ring has been designed. It has a constant thickness of 40 cm and a varying width and divides the roof deck into two parts: one located inside the ring and the other spanning between the ring and the arches. For the part inside the roof it was considered unfeasible to compute the stresses on the basis of the theory of shells. For the design in one direction it was conceived as a vault and in the other as a truss. Consequently and in order to obtain clear static conditions it was prestressed in one direction by means of 25-tons-prestressing tendors at 85 cmcenters. The concrete is 7 cm thick. (fig. 18).

REFERENCES

1. Haas: Prestressed and Precast Shells, 2nd Symposium on Shell Roof Construction, Oslo 1957

2. Blumfield: The Combination of Shells and Prestressing, Symposium on Concrete Shell Roof Construction, London 1952.

3. Guyon: Conférence à l'Université de Rome, 1951.

4. Haas: Design and Calculation of Northlight Shells in Prestressed Concrete for 2 x 131 ft. continuous, De Ingenieur 1956. (In Dutch with summary in English).

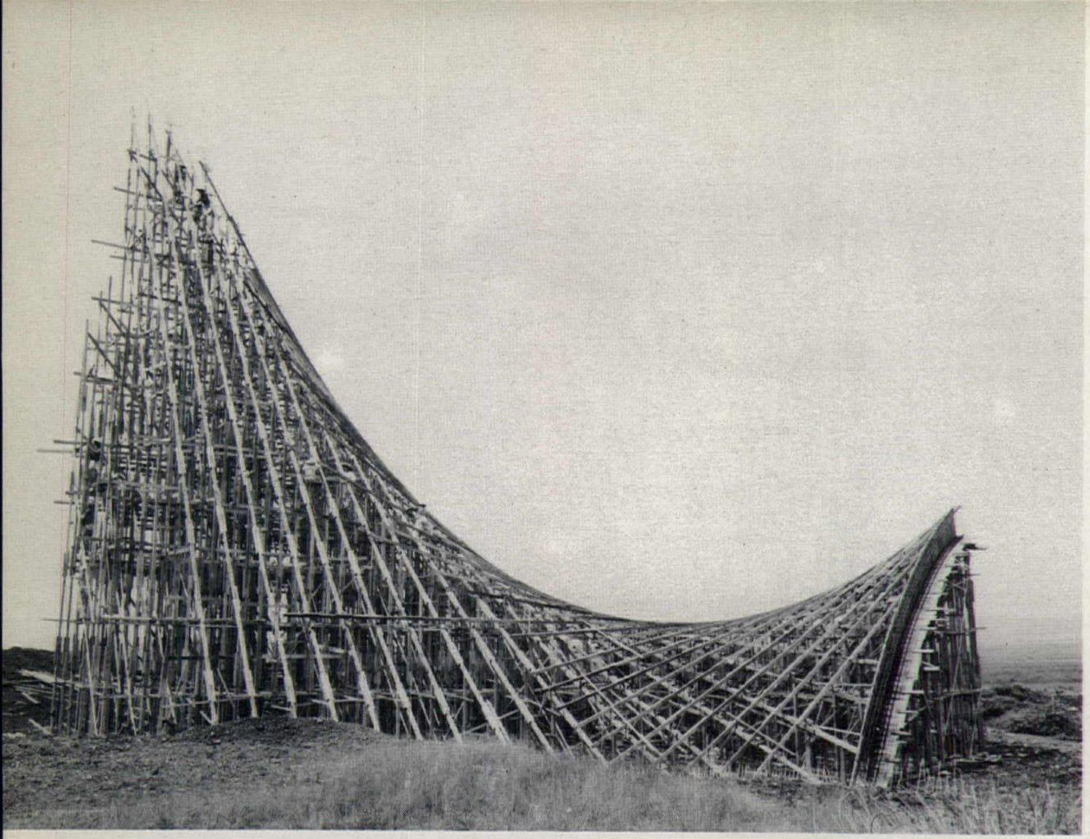
5. Haas: Research on a Prestressed Concrete North-light Shell Structure, 2nd congress F.I.P. Amsterdam 1955.

6. Schmausser: Berechnung zylindrischer Schalendächer unter Längsvorspanning nach der Methode des Stellvertretendes Faltwork, Die Bautechnik, Febr. 1957.

7. Haas: Prestressed Shells and Precast Units, Proceedings World Conference on Prestressed Concrete, S.F. 1958.

8. Timoshenko and Goodier: Theory of elasticity 1951.

9. Koepcke: The Berlin Congress Hall, special Issue of the 3rd International Congress on Prestressed Concrete, Deutsche Beton Verein 1958.



REINFORCED CONCRETE SHELLS

FELIX CANDELA, *Spanish architect practicing in Mexico, has achieved wide reputation for his imaginative work with concrete shells and for his methods of mathematical analysis.*

This article appeared in Spanish in the Mexican journal Espacios, January 1959, and was translated for this publication by Dr. George W. Poland, Head, Modern Language Dept., N. C. State College.

The construction of curved laminar structures of reinforced concrete has experienced a notable increase in the last ten years. Judging from the considerable number of projects appearing in professional journals in which such structures are employed, the growing interest that its constructive and esthetic possibilities has awakened is clear. Nevertheless, the interest of architects and engineers in this type of construction has not been preceded, nor even accompanied, by the discovery or natural evolution of trustworthy methods for the stress analysis of such structures and even less of methods sufficiently simple so as to regulate their application so that they can be used by the nonspecialist engineer. Both the design of such structures and the previous choice of their form and disposition are generally made in a somewhat arbitrary manner because of the lack of a body of doctrine available for general knowledge. This obliges those who have devoted their efforts to the investigation and construction of laminar structures to try to examine publicly the present state of the problem of their design and especially to circulate knowledge as to the reasonable possibilities of execution of the different forms known and the conditions that must be fulfilled for new forms that may be proposed.

Among the various points to which I will be referring in the course of this article, the following should be pointed out:

a) The problem of form in relation to structural behavior and the present possibilities of calculation.

b) Limitations imposed by the necessity of calculating the stresses previously and by the economic condition that the cost of the calculations must not exceed a small percentage of the total cost of the structure; the practical impossibility of making an exact analysis for all the possible cases of load.

c) Influence on the project of the size or scale of the structure.

d) Difficulties and inconveniences of analysis by means of tests made on reduced models.

Under the name of "shells" are usually grouped a series of structural forms whose structural behavior differs essentially however, from one type to another in accordance with the shape of the surface. An adequate definition of what is to be understood by a so-called "shell" would help considerably to avoid confusion and constitutes the first step in any explanation of a general nature.

It would be clearer, in my judgment, to use the general name of "laminar structures" for all those structures in which the thickness is very slight in relation to the other two dimensions and to restrict the use of the term "shell" structures to those laminar structures which would be capable of working, under normal load conditions, with membrane stresses only; that is to say, without bending stresses. Let us call "membrane stresses" those that are uniformly distributed in the laminar thickness and act parallel to the plane tangent to it at any point.

One essential condition to prevent bending is that the surface which constitutes the shell be of a double curvature; that is to say that it has a geometrically immutable form as long as considerable lengthening or shortening is not induced. With the relatively inextensible materials that are used in construction, with reinforced concrete particularly, such longitudinal variations are possible only when the membrane stresses (of compression or tension) reach very high values which exceed the elastic limit of the material. This means that if it is possible to analyze the membrane stresses that are produced in a shell structure and if the resulting stresses do not exceed admissible values, bending that would have to be accompanied by change of form or of curvature of the surface cannot appear; and it is not necessary, therefore, to go back to the general theory of bending for the study of the structure. It is curious to observe the fact that the majority of theoretical studies do not seem to take into account the properties or characteristics of the geometric shape nor the practical impossibility that there may co-exist in the elastic range bending and membrane stresses. So that bending may occur in the elastic range the membrane stresses must have surpassed the elastic limit and be acting in the plastic range; and, therefore, the whole precious mathematical artifice of the general theory of bending falls apart.

An intuitive demonstration of the preceding can be obtained by considering a revolution dome with loads parallel to its axis and symmetrical around it. So that bending or changes of curvature in the meridians can appear, for example, it is necessary to shorten certain parallels and lengthen others. Observe also what happens upon trying to produce a dent in a sphere by applying a concentrated pressure at some point. It is necessary in this case for the surface immediately around the spot of application of the pressure to lengthen concentrically to allow a surface of greater area to pass through. This is perfectly possible in a rubber ball which is made of a very extensible material but cannot occur in one of concrete until the tensile stresses along its circumferences have exceeded the elastic limit of the material.

In summary it can be said that a surface of double curvature, completely flexible but not extensible, has an immutable form under the action of any loads. There is no purpose, therefore, in trying to extend the funicular concept to surfaces, attempting to shape them in accordance to the distribution of permanent loads and giving rise to surfaces called **"velarias"** or **"anti-velarias"** (the shape of sails). An arch, which is a lineal structure, can only work with direct or membrane stresses, without bending, when its form coincides with the funicular of the loads, but a suface structure of double curvature develops only membrane stresses under any system of loads. This is a property that nature makes use of since all natural shells, and particularly those of stony materials like an egg or a snail, use forms of double curvature. Since it is not necessary in this case for the surface to have any resistance to bending, its thickness can be reduced to the minimum economically or practically possible,

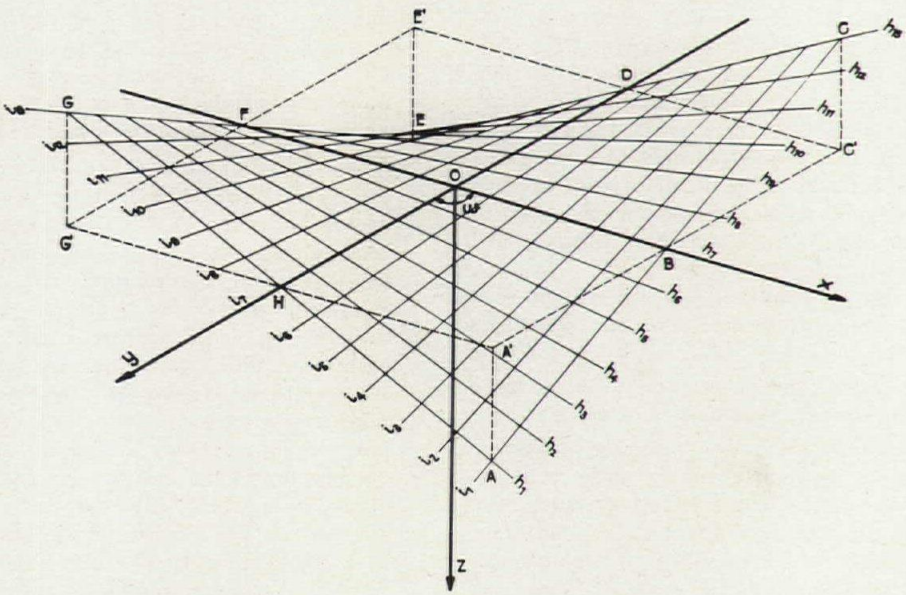


Fig. 1. ***Generation diagram of a hyperbolic paraboloid.***

obtaining in this way the two fundamental advantages of this type of structure: reduction of its own weight and the possibility of adapting itself structurally to cases of unforeseeable loads without ceasing to behave as a membrane. Keeping in mind that such behaviour is fundamentally more economical than that of bending, since stresses are shared uniformly in the section, double curvature surfaces are the most interesting from the structural point of view and the only ones that should be called **shells.**

Another group of curved surfaces which are commonly used in construction are those of simple curvature: cylinders, cones, and in general all developable surfaces which are, as their name indicates, obtained by bending a flat surface. It is obvious that in this case the only thing that opposes the modification of the original curve is the rigidity of the surface to bending. If we consider that the surface is flexible, its form is totally unstable and small variations of the load pressures that act upon it produce deformations or changes of curvature in its cross section. In order to stabilize the form, it is necessary to introduce elements foreign to the surface itself such as diaphrams or rigid arches at relatively short intervals. Nature gives us a very clear example of the necessity for this process in bamboo canes with their rigid diaphrams at short intervals. When rigid elements are placed at more considerable distances, such as is the case in long barrel vaults, the changes of form in the cross sections at the central part of the span between stiffeners can only be counter-arrested by the resistance to bending of the surface itself since the effect of torsional rigidity of the vault disappears at a short distance from the stiffeners.

The fundamental difference between the structural behavior of surfaces of simple curvature and double curvature can be observed very clearly by noting the manner of failure in both. The first fail by bending and the second fail by lengthening.

These considerations show that developable surfaces not having a geometrically stable form cannot be considered as membranes nor can their stresses be calculated as the classic German theory precognizes, which has recently been sanctioned offi-

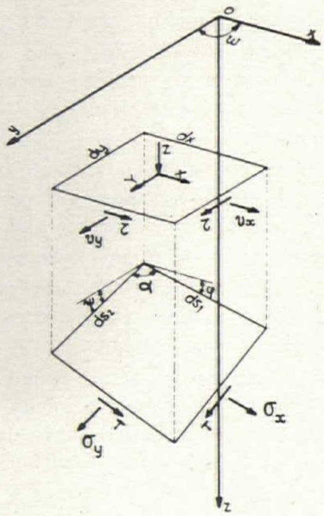


Fig. 2. *External forces and internal forces which act on a surface element.*

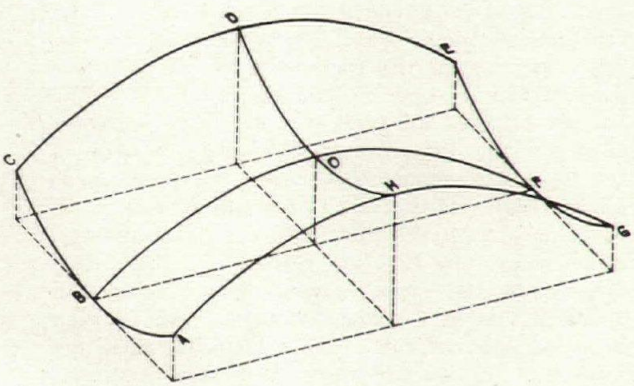


Fig. 3. *Hyperbolic paraboloid limited by principal parabolas and considered as a translation surface.*

cially with the publication of a manual based on it by the A.S.C.E.

In certain cases, as in short barrel vaults having an antifunicular directrix of the permanent loads, the structure can function occasionally as a membrane for the given state of loads; but as soon as live loads appear with different distribution, this unstable balance is changed and bending of the surface begins. In reality these short vaults function in the same way as traditional stone vaults in which the anti-funicular or pressure line of the loads is applied within the central nucleus of each voussoir. Since, in these stone vaults, their own weight was enormous, the possible live loads were of a much smaller order of magnitude and their intervention did not alter fundamentally the original pressure line. On the other hand, in modern concrete vaults of insignificant thickness, their own weight is comparable and, at times, less than live loads (wind, snow, etc.). The action of the latter causes, therefore, considerable alteration of the pressure line which emanates totally from the section causing bending of the laminar and the necessity to increase its thickness. For large spans it is not possible to increase the thickness as much as the moments require, and it is necessary to arrange rigid arches at distances to be determined empirically.

In long barrel vaults which function structurally like hollow circular beams, transverse bending always appears in the sections removed from the supports as a result of the fixing of the static balance between the loads that act in a transverse fillet and the difference of shear stresses between the two cross sections that border this fillet. As is known, the loads that act in each fillet have to be in balance with the difference of shear stresses. Nevertheless, the first are vertical and the second tangent to the cross section. Consequently, the moments that both systems of forces produce, even being of opposite sign, cannot be equal since the lever arm of the first force is greater than that of the second. This bending can be lessened by means of edge beams or by considering the continuity between contiguous vaults, but they exist anyhow and must be taken by the lamina, which must be given a substantial thickness.

This situation can be appreciated even better in prismatic structures formed by folded slabs which act like deep beams. It is evident that, considering their edges as supports for the slabs in the transverse direction, these slabs must work by resisting bending between such supports solicited by the normal load components to the slab.

The simplest surface structure is the horizontal flat slab, but it can never be considered as a shell since its structural action depends exclusively on its capacity to resist bending. Starting from this form we find a gamut of structures in which part of the load is transmitted to the supports by direct or membrane stresses, but in which another part of the load is transmitted by bending of the laminar itself, until we reach those structures formed by doubly-curved surfaces in which the entire load is transmitted to the supports by membrane stresses without bending appearing until the stages immediately previous to failure are reached.

The progression between both cases can be established in the following way:

a) Flat horizontal slabs. (Bending stresses exclusively).

b) Prismatic structures or folded slabs. (Mixed membrane and bending stresses).

c) Conical vaults, barrel vaults, and developable surfaces in general. (Mixed membrane and bending stresses with the first bending to predominate).

d) Double curvature surfaces. (Membrane stresses only, provided the arrangement of the structure and its supports is correct and the values of the stresses do not exceed admissible limits.)

Double curvature surfaces which give way to the so-called shell structures are classified according to their form into two large groups:

1) **Synclastic** surfaces, also called **elliptical** through the form of the equation which represents them, in which the two principal curvatures in each point are directed in the same direction. The most obvious example of this type is the spherical cal dome.

2) **Anticlastic** or **hyperbolic** surfaces in which both principal curvatures are directed in opposite directions as in a horse saddle. Perhaps the clearest and best known example is the hyperboloid of one

leaf which is obtained by twisting a cylinder formed by threads held in two circles at the base giving rise to a figure similar to a "diavolo."

Both groups of surfaces share the property of being able to work with membrane stresses exclusively which makes them very appropriate—considering them from the purely generic and abstract point of view—for covering large spaces with a minimum of material and therefore of weight. However, for the use of such surfaces in construction, we cannot disregard a series of circumstances which depend upon the site, such as the climate, the degree of evolution of the local construction industry, which can be more or less mechanized, the cost of manual labor, economic material, etc., that can influence the choice of the form. From a more universal point of view, the decisive factor in selecting a structural form is the existing state of analytic technique which must serve us for the prior investigation of stresses in the structure.

Among all these circumstances, the first relate principally to cost, and since that is not a decisive factor in many cases of sumptuous buildings, they do not constitute a rigorous limitation for the use of certain forms. On the other hand, the last point, the real possibilities of calculation, does definitively limit freedom in the use of arbitrary forms since long years of believing, more or less illusorily, that we know what happens in a structure obliges us to try to determine analytically the probable value of forces that act in it before deciding to build it.

In view of the complexity of the mathematical problem that the attempt at analytical calculation usually sets up, there are many who advocate the use of tests on reduced models hoping to find in them a comfortable method of eluding difficulties that appear insuperable and of avoiding the painful task of thinking. Those who propose this forget the limitations and the real scope of such a procedure.

In the first place the models must be made with the same material as the final structure, which means making them on a rather large scale at a high cost. In the second place, in order for a test to be successful we must know beforehand what we are looking for. This may seem a platitude, but the fact is that in many universities tests are being made with shell models, and it is very rare that the people who carry them out can explain just what the real

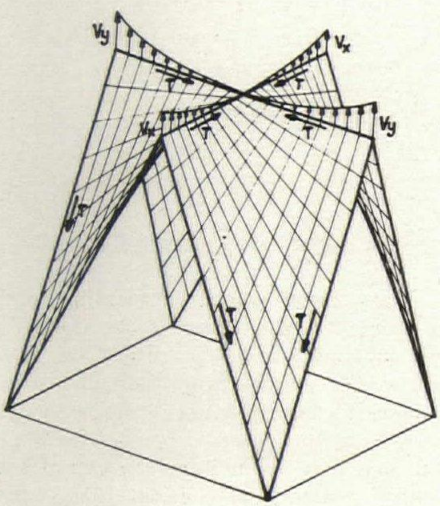
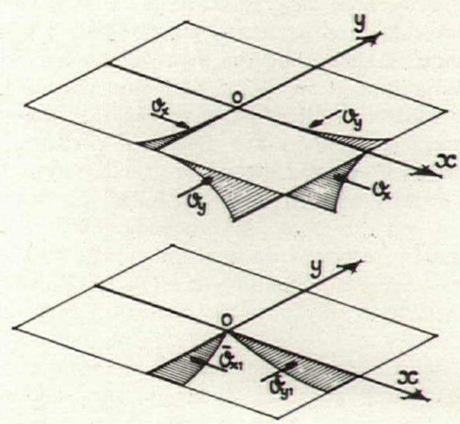


Fig. 4. ***Diagram for a structural study for La Virgen Milagrosa Church.***



purpose of the test is. This waste of effort is only justified by "the vice of working for work's sake" of which Ortega speaks. It is a question of trying to do "something", whatever it may be, so long as it has a complicated appearance and many electric wires with which one can deceive himself into thinking that he is doing something "scientific."

In reality the model can only prove approximately that the working conditions are the same as we have supposed. Keep in mind that what is important generally is not the stresses in the lamina itself, which usually have insignificant values, but the forces that the latter transmit to the beams or to the supports. Without knowing at least the order of magnitude and the probable direction of such forces, it is not possible to give dimension to the edge beams or to decide whether it is possible to suppress them. But the size, the arrangement of the reinforcement and, consequently, the deformations of these have decisive influence on the distribution of stresses in the surface and on the behavior of the structural whole.

I cannot refer here in detail to the complications which a change in scale causes, which means loading the model with weights that are not proportional to those of the real structure, nor can I deal with the difficulty of interpreting results with the lack of precision in measurement of the deformations and the uncertainty in the relation between these and the stresses. Our only recourse is to suppose that the materials are perfectly elastic and that their behaviour is such that the stresses do not exceed the elastic limit at any point, both suppositions being patently false. It all adds up to the fact that tests only serve as an auxiliary means of certifying the correctness of our calculations or previous hypotheses but in no way as a basis for an initial analysis.

There remains then, in my judgment, no other path than to attempt an analytic calculation of the stresses and, unfortunately, from a practical point of view, this is only possible in certain very simple cases.

The calculation consists of the determination of the membrane stresses which is carried out by means of a process in which only statics is involved, without having to take into consideration elastic properties of the materials which always have a hypothetical and uncertain character. Such a process consists of the establishment of the equilibrium conditions, according to these axes in space, of a differential element of surface in which external and internal forces applied on the faces of the element act. We obtain, thus, a system of three differential equations in partial derivatives whose general expression in the most usable form is given by Pucher. (A. Pucher. "Ueber den Spannungszustand in doppelt gekrummten Flachen." BETON UND EISEN, Berlin, V. 33, No. 19, Oct. 5, 1934, p. 298).

Keeping Fig. 2 in mind and if the surface is represented by the equation

$$z = F\ (x,y) \tag{1}$$

in a system of co-ordinated axes, birectangular in that the angle **xoy** can have any value and the angles **xoz**, **yoz** are right angles, the following equations result:

$$\begin{aligned} \frac{\delta \upsilon_x}{\delta_x} + \frac{\delta \tau}{\delta y} &= -\ X \sin\omega \\ \frac{\delta \tau}{\delta_x} + \frac{\delta \upsilon y}{\delta \gamma} &= -\ Y \sin\omega \\ r\upsilon_x + t\upsilon y + 2s\tau &= (pX + qY - Z) \sin\omega \end{aligned} \tag{2}$$

in which the significance of the different letters is:

$$\begin{aligned} p &= \frac{\delta Z}{\delta X} \qquad q = \frac{\delta Z}{\delta y} \\ r &= \frac{\delta^2 Z}{\delta x^2} \qquad t = \frac{\delta^2 Z}{\delta y^2} \qquad s = \frac{\delta^2 Z}{\delta x \delta y} \end{aligned} \tag{3}$$

which are the partial derivatives of equation (1).

X, Y, Z are the components, according to the co-ordinate axes, of the external forces (loads) measured per unit of surface in projection upon the plane xy. υ_x, υ_y, τ are the projections upon the plane xy of the real stresses δ_x, δ_y, T respectively, both systems representing stresses per unit of longitude; that is to say, they are unitary stresses although it is generally necessary to consider them multiplied by the thickness of the surface.

Both systems of stresses are tied together by the following expressions:

$$v_x = \sigma_x \sqrt{\frac{1+q^2}{1+p^2}} \qquad v\gamma = \sigma_y \sqrt{\frac{1+p^2}{1+q^2}} \qquad \tau = T \tag{4}$$

The equations (2) constitute a system of linear partial differential equations, which it is necessary to solve for each form of surface and for each case of load. Its solution gives us in each case the values of the membrane stresses v_x, v_y, τ at each point, but these values on being obtained by means of integrals are effected by arbitrary constants of integration; that is to say, we do not obtain fixed values, but laws of variation from some points to others. With the object of determining unequivocably the said values, it is necessary to fix them at some points which, generally, correspond to the edges which are the places where we can know or fix the forces that the edge beams are capable of resisting. This property permits us a certain degree of liberty in choosing the boundary conditions, which is the same thing, the way of supporting the structure. It is a question in the final analysis of an indeterminate problem of the same type as those with which we are accustomed to deal when studying any hyperstatic or redundant structure which has more possibilities for support and, consequently, for equilibrium than those that the conditions of statics require. It is necessary, in these, to determine the support reaction previously in order to be able to study the stresses on the inside of the structure. The elastic method of determining these reactions consists in finding the conditions in which the structure does a minimum of work since this seems to be a general law in nature. For it, we have to suppose that the structure is purely elastic. That this procedure is admitted officially by the codes in use does not mean that the support reaction cannot be selected by many other methods, including fixing them arbitrarily, using as a basis the real properties of plasticity of the construction material. If we consider, for example, a flat rectangular slab with loads normal to its plane, it is evident that equilibrium is possible in many ways. We can consider it fixedly clamped on its four sides, or simply supported on any of them, or on all, or finally transform it into isostatic, by considering it as fixedly clamped on one of its sides and without support on the others. Balance is possible in all these cases, but naturally the stresses that are caused in the slab are different for each one of them.

The analytical problem of shells is complicated by the consideration of the edge conditions because we have to keep them in mind in solving the system of equations (2).

In the general case, the problem consists in finding a certain function whose second derivatives are the stresses that satisfy the edge conditions and equations (2) at the same time. It is a problem that can also be approached by means of developments in series or by finite differences, both procedures long and tedious whose cost in time and in money is prohibitive in the majority of cases.

The problem has been solved and tabulated for certain very simple forms of structure also having very simple load systems as, for example, for spherical surfaces limited by vertical planes, with vertical loads, which give rise to the so-called "kerchief domes."

On surfaces of revolution with loads parallel to the axis and symmetrical in relation to it, the problem can be attacked in a very simple form by making use of the total symmetry of the structure. The solutions are very well known and appear in the majority of books dealing with these themes as well as in general treatises.

Aside from these well known cases, the general method is practically unapproachable until the use of electric computers and analogical machines becomes popular and cheaper. At present I believe that the method can be used only for the calculation of certain metallic shells used in aviation or ballistics and perhaps in that of dams that act as semi-membranes. In the first case the aeronautical industry can spend great sums in the prior investigation of prototypes that are to be afterwards mass produced and sold at a very high price. In the second, the high budget of a great dam can allow the use of a minimum part of the money for covering the enormous cost of an investigation of this type.

In ordinary construction, however, we cannot allow ourselves such luxury. We continue to propose a different solution for each problem, and this condition has become aggravated even more in recent times by the excessive desire for originality which, at present, characterizes architecture. A good solution is usually rejected merely because it has been used previously by another. Only unused solutions

are acceptable for a truly modern architect. But very few are disposed to sufficiently immerse themselves in the problem to be able to propose reasonably simple solutions and, of course, no one is in position to defray the expenses that a careful analysis of a whimsical form requires. The result of this situation is that a great number of projects in which arbitrary forms are proposed remain on paper or are built in an absurd manner at considerable waste of material by attempting to increase the safety coefficient by means of an increase of mass, without taking into account the fact that both increases usually vary in inverse proportion, as Maillart has already carefully observed and as experience shows us. There exists an argument in favor of the anti-economical solution, and it is that the owners always prefer to spend more money on the job if this expenditure is justified by a greater quantity of material which, after all, remains in the building and can be appreciated visibly, rather than to pay something more than the strict minimum for prior research whose result is unfathomable.

But let us return to the problem of shells after this digression.

I have tried to show by means of considerations of an intuitive nature that shells must have a double curvature. It is obvious that as long as new low cost plastics having suitable characteristics are not developed, the material appropriate for the construction of shells is reinforced concrete because of its low cost, the ease with which its basic materials can be found at any spot, and its moldability to give the desired form. This form is obtained by means of the prior erection of molds, and at present these can be constructed only by using wood. The use of metallic molds can only be justified economically when the conditions of the problem permit their reuse a large number of times. It we admit this fact and if we accept also that economy is a fundamental condition of construction, we easily arrive at the elimination, or practical limitation, of the use of **synclastic** or elliptical surfaces that require the preparation of numerous curved forms upon which it is necessary to curve the staves by following a very costly procedure.

The considerations that I made before, as far as calculation is concerned, also eliminate all those surfaces of arbitrary definition which do not have a relatively simple mathematical expression, except for clearly insignificant spans in which exact calculation can be omitted, if a reasonable interpretation of the regulations permits it.

Among the **anticlastic** surfaces of simple geometric definition there exists a group of so-called **ruled** surfaces which possess the property of being generated by straight lines that move along the surface following certain laws. An example is the **conoid** which is produced by a straight line that is moved, being supported on any two curves and remaining parallel to a plane called the **director plane.** It is evident that this property can be used advantageously in the erection of the centering by placing the pieces of wood along the so-called generator lines. The cases in which the surface has two systems of generator lines are still preferable from the constructive point of view because then not only the boards but also the sustaining joists can be straight. This leads us to the consideration of only two geometric surfaces: **the hyperboloid of a leaf** and the **hyperbolic paraboloid.** Both have a clear and simple equation since they belong to the group of **quadrics** and possess two systems of rectilinear generators.

We can even eliminate the first of them because, except in the case of the hyperboloid of revolution with vertical axis, the solution of its membrane stress equations presents mathematical problems that are practically unapproachable, coming thus by elimination to the consideration of the only surface which in the present state of constructive and analytic technique possesses all the desirable conditions for a shell.

Even at the risk of lengthening this article excessively, I wish to examine briefly the geometrical characteristics of the hyperbolic paraboloid and show the reasons for which its calculation is the only one possible under reasonable conditions of time and effort.

Let us assume two straight non-parallel, non-intersecting lines, HOD and ABC, (Figure 1) crossing themselves in space, which will be called temporarily **directrices.** Straight lines h_n which cut the two directrices, being at the same time parallel to any plane xOz, called director plane, define the surface called hyperbolic paraboloid and constitute the **first system of generators** of it. The two directrices define, in their turn, a second director plane, **yOz**, parallel to them. The surface can also be considered as formed by a second system of generators, i_n, parallel to this second director plane and cutting all the generators, h_n, of the first system.

The hyperbolic paraboloid contains, therefore, two systems of rectilinear generators, h_n and i_n. Each system is parallel to a director plane and both director planes from any angle ω. Any point of the surface is the intersection of two straight lines contained in it. Each generator of a system cuts all those of the contrary system but none of those of its own system since they are located on parallel planes. (Although it would be more proper to say, in mathematical language, that all the generators of one system intersect the **improper line** of their director plane.)

The equation of the paraboloid is usually referred to a Cartesian or trirectangular system of coordinate axes formed by: the intersection of both director planes as axis z and the bisectors of angle ω which are tangent to the surface as axes x and y. In this case we obtain the normal equation

$$z = ax^2 - by^2 \qquad (5)$$

However, many advantages are gained by using the equation that results from considering as coordinated axes **xy** the two generators HOD, FOB which pass through the vertex of the paraboloid and are contained in a plane perpendicular to the line of intersection of the two director planes. This line is taken as axis **z.** In this birectangular coordinates the equation of the surface is reduced to:

$$z = k\,x\,y \qquad (6)$$

in which **k** is a constant that depends on the curvature or warping of the paraboloid

$$k = \frac{AA'}{OB\,.\,OH}$$

Observe that the equation of the second degree (6) is the simplest that can exist tying the three variables x, y, z.

The angle xOy can have any value ω, but the angles xOz and yOz are right and the system of coordinate axes is birectangular. When the director planes are perpendicular among themselves ($\omega = 90°$), the paraboloid is called **equilateral.**

Plane sections parallel to axis z are parabolas or straight lines. Of these sections the ones that are, moreover, parallel to the bisector planes of the dihedral director ω are called **principal parabolas** and are curved respectively upward (GOC) and downward (AOE), hence the surface is **anti-clastic.**

All the other plane sections, not parallel to axis z, are hyperbolas, or their degeneration, in two straight lines.

As a translation surface (Figure 3) the paraboloid can be considered engendered by a principal parabola ABC which moves parallel to itself along the inverse principal parabola BOF. Consequently, the surface has two systems of parabolic generators. Each system is composed of identical parabolas situated on parallel planes.

All these known geometrical properties of the hyperbolic paraboloid are very useful for the erection of the formwork or centering and for the design of the shape.

By applying the system of equations for membrane equilibrium (2) to the equation (6) for the paraboloid and keeping in mind that, in this case:

$$p = \frac{\delta_z}{\delta_x} = ky$$

$$q = \frac{\delta_z}{\delta_y} = kx$$

$$s = \frac{\delta^2 z}{\delta\chi\delta_y} = k$$

$$r = \frac{\delta^2 z}{\delta X^2} = 0$$

$$t = \frac{\delta^2 z}{\delta y^2} = 0 \qquad (7)$$

we obtain the following system of differential equation:

$$\frac{\delta \upsilon_x}{\delta X} + \frac{\delta \tau}{\delta y} = -\,X \sin\omega \qquad (8a)$$

$$\frac{\delta \tau}{\delta X} + \frac{\delta \upsilon_y}{\delta y} = -\,Y \sin\omega \qquad (8b)$$

$$\tau = \left[\frac{X}{2}y + \frac{Y}{2}x - \frac{Z}{2k}\right]\sin\omega \qquad (8c)$$

Since the third equation of system (8) is an algebraic expression, the system is integrable directly, which constitutes a unique property—of invaluable value—of the hyperbolic paraboloid.

When the components of the load (X, Y, Z) are continuous and derivable functions, the system of equations (8) is solved in the following manner. Equation (8c) is differentiated with respect to y. By substituting this differential in equations (8a) and integrating it with respect to x, we obtain the expression of unitary stress υ_x. Analagously, by differentiating (8c) in respect to x, substituting in (8b) and integrating this with respect to y, we obtain υy.

For unusual cases of load the integration of (8a), (8b) can offer mathematical difficulties. However, in the usual cases of load, integration presents no difficulties. For example, for the dead weight of a surface of uniform thickness, in the general case of loads not parallel to axis z (or what is the same thing, of paraboloids with the axis z not vertical), the solution is

$$\tau = \left[\frac{1}{2} yX_1 + \frac{1}{2} xY_1 - \frac{1}{2k} Z_1 \right] \sqrt{\phi} \qquad (9a)$$

$$\upsilon_x = \left[\frac{1}{4} (\cos\omega Y_1 - 3X_1)x + (\frac{5}{4} \cos\omega X_1 - \frac{1}{2} Y_1$$

$$+ \frac{3}{4} \cos^2\omega Y_1)y - \frac{1}{2k} \cos\omega Z_1 \right] \sqrt{\phi}$$

$$+ \left[(- \frac{5}{4} k \sin^2\omega X_1 \right.$$

$$- \frac{3}{4} k \cos\omega \sin^2\omega Y_1)y^2 + (\frac{1}{2} Z_1 \sin^2\omega)y$$

$$\left. - \frac{1}{4k} \sin^2\omega \ (3X_1 + \cos\omega Y_1) \right]$$

$$\mathrm{Log_n} \left[\frac{kx - ky \cos\omega + \sqrt{\phi}}{\sin\omega \sqrt{1 + k^2y^2}} \right] + f_1(y). \qquad (9b)$$

$$\upsilon_y = \left[\frac{1}{4} (\cos\omega X_1 - 3Y_1)y \right.$$

$$+ (\frac{5}{4} \cos\omega Y_1 - \frac{1}{2} X_1 + \frac{3}{4} \cos^2\omega X_1)x$$

$$\left. - \frac{1}{2k} \cos\omega Z_1 \right] \sqrt{\phi}$$

$$+ \left[(- \frac{5}{4} k \sin^2\omega Y_1 \right.$$

$$- \frac{3}{4} k \cos\omega \sin^2\omega X_1) x^2 + (\frac{1}{2} Z_1 \sin^2\omega) x$$

$$\left. - \frac{1}{4k} \sin^2\omega \ (3Y_1 + \cos\omega X_1) \right]$$

$$\mathrm{Log_n} \left[\frac{ky - kx \cos\omega + \sqrt{\phi}}{\sin\omega \sqrt{1 + k^2x^2}} \right] \qquad (9c)$$

in which: X, Y, Z, are the components of the load measured per unit of real surface, $\phi = \sqrt{\sin^2\omega + - k^2x^2 + k\gamma^2 - 2k^2x\gamma\cos\omega}$ f_1 (y), f_2 (x) are arbitrary functions of integration which are determined by the boundary conditions

Equations (9) are simplified, obviously, depending on the conditions of the problem. If, for example, axis z of the paraboldoid is vertical, $X_1 = Y_1 = 0$ and much shorter expressions result. When the paraboloid is equilateral ($\omega = 90°$, $\cos\omega = 0$). If, however, in addition to these two simplifications, the inclination and curvature of the surface are sufficiently small so that we can consider approximately that the load is uniformly spread per unit of projection over the plane xy, we shall have $Z = g =$ constant, and expressions (9) are reduced to

$$\left. \begin{aligned} \tau &= - \frac{g}{2k} = \text{constant} \\ \upsilon_x &= f_1 \ (y) \\ \upsilon_y &= f_2 \ (x) \end{aligned} \right\} \qquad (10)$$

Keep in mind that this drastic simplification constitutes an approximate solution whose use is only licit in certain cases in which in the judgment of a experienced and conscientious designer it is fully justified. It is necessary to insist repeatedly on this warning since, unfortunately, the erroneous belief that this approximate solution is valid for any form and disposition of the paraboloid is very widely spread. The source of this error which is so general is to be sought in a French book (L. ISSENMANN PILARSKI, **"Le Caloul Des Voiles Minces En Beton Arme"**. E. D. DUNOD. Paris, 1935), in which it is stated in a vague manner that the paraboloid is a

shell of equal resistance (or what is the same thing, that shear stresses τ are identical on any point of the surface) without specifying clearly the type of load for which this is the case. The few articles on the calculation of these surfaces that have appeared since have done nothing to clear up this concept since they are in general informative articles without any critical judgment based on the French texts which are rather obscure.

From the observation of equations (9) the following consequences are obtained:

a) The values of τ are completely defined from the beginning and, consequently, the boundary conditions do not influence them.

b) The values of δ_x and δ_y (and consequently of σ_x and σ_y) are indeterminated proving thus the hyperstaticity of the membrane state of stresses in a double curved shell. Once their value is fixed at any point (which is equivalent to giving a certain value to integration functions f_1 (y) and (f_2 (x).), the stresses remain determined along the generators that meet at the point. These considerations allow us to give certain general rules for the equilibrum of structures formed with hyperbolic paraboloids:

1. A surface limited by four staight generators can have two contiguous sides free from oblique stresses, but the two opposite sides must have complete support, that is to say; supports capable of taking forces contained in the planes tangent to the paraboloid along the border considered.

2. Any straight edge is subjected to compressive tensile forces resulting from the sum of the unitary shear stresses τ that are left unbalanced upon limiting or cutting the surface along such a straight edge. When the supports of the structure are arranged in such a manner that these are compressive forces, it is necessary to arrange a rib along this edge and its own weight gives rise to deformation or deflections which must be kept in mind and try to avoid. As a general rule it is necessary to arrange secondary vertical supports capable of supporting the additional weight of the edge rib.

3. The normal stresses and tangential stresses on an edge not parallel to the generators are obtained by means of the formulas:

$$\sigma\beta = \sigma x \frac{\sin^2\beta}{\sin\alpha} + 2\tau \frac{\sin\beta \sin(\beta - \alpha)}{\sin\alpha} + \sigma_\gamma \frac{\sin^2(\beta-\alpha)}{\sin\alpha} \qquad (11a)$$

$$\tau\beta = -\sigma_x \frac{\sin\beta \cos\beta}{\sin\alpha} - \tau \frac{\sin(2\beta-\alpha)}{\sin\alpha} - \sigma_y \frac{\sin(\beta-\alpha)\cos(\beta-\alpha)}{\sin\alpha} \qquad (11b)$$

in which α is the angle formed by the two generators which pass through the point (α is variable from one point to another), and β is the angle which the tangent to the cut forms with the positive part of the generator of system χ which passes through the point. Since σ_x and σ_y are indeterminate or variable, it is possible for $\sigma\beta$ and $\tau\beta$ to adopt any value which pleases us, including zero. In the later case the corresponding values of σ_x and σ_y are:

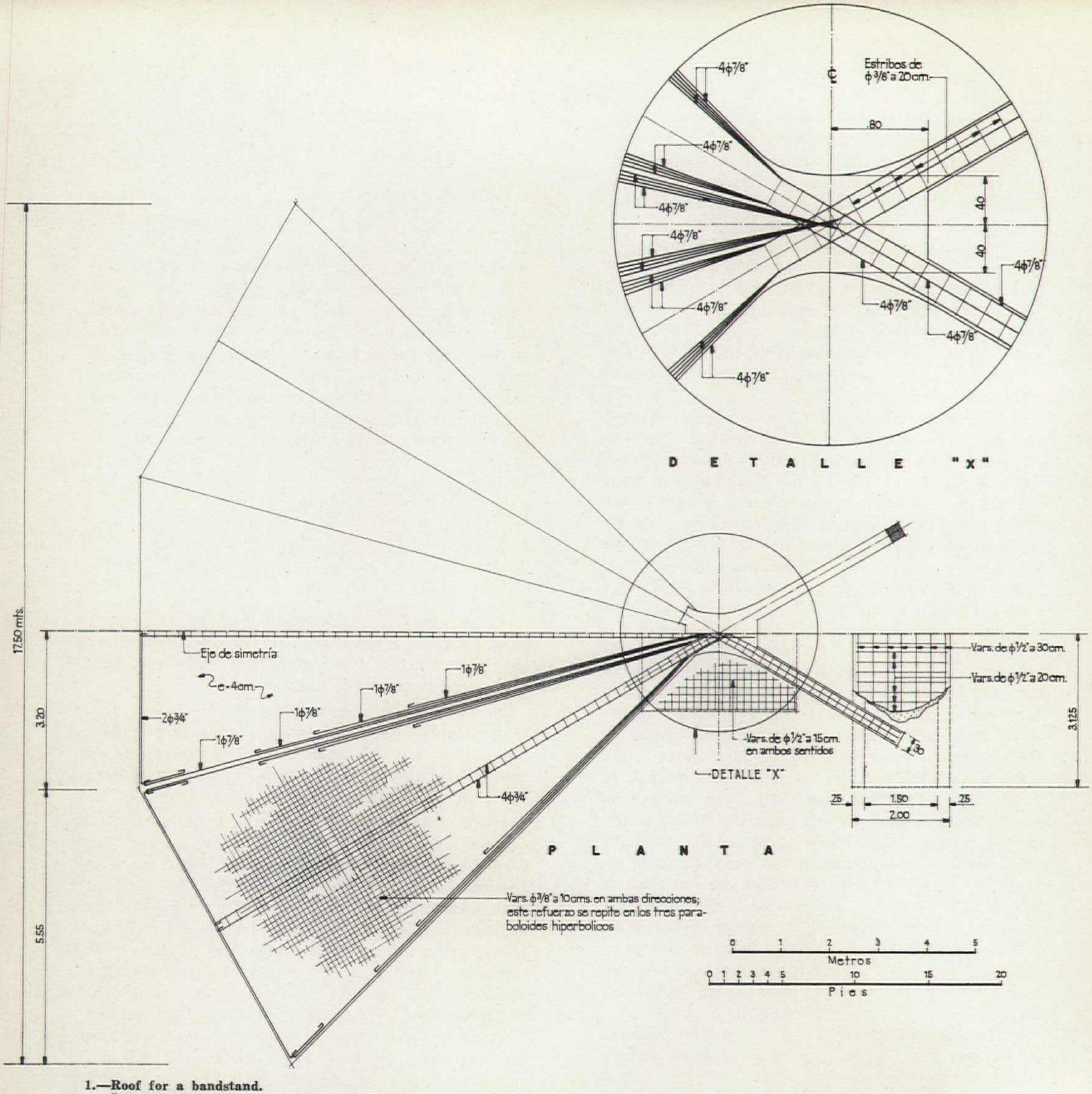
$$\sigma\gamma = -\tau \frac{\sin\beta}{\sin(\beta-\alpha)} \qquad (12a)$$

$$\sigma\chi = -\tau \frac{\sin(\beta-\alpha)}{\sin\beta} \qquad (12b)$$

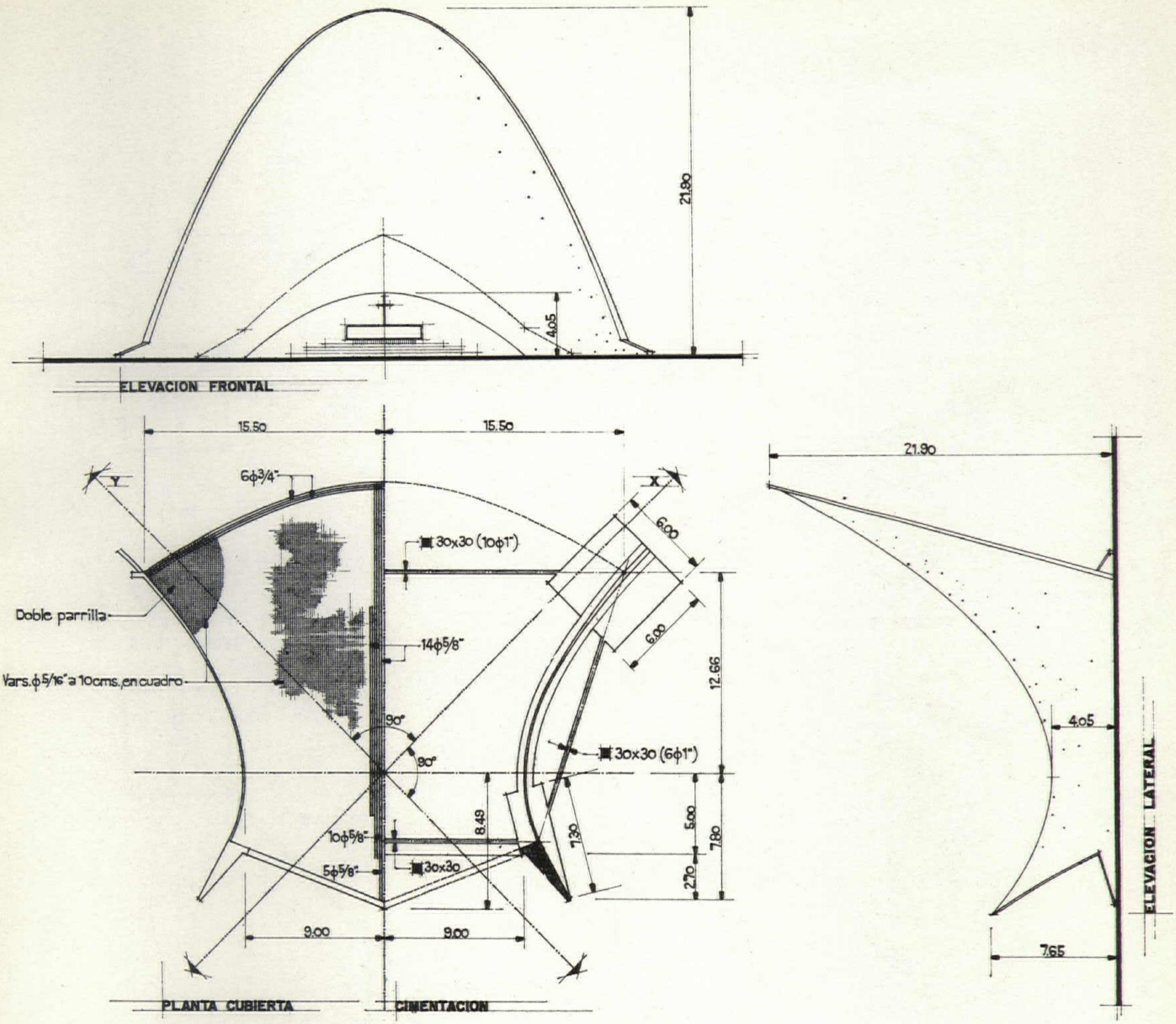
Naturally, on fixing these values at one point, they remain automatically fixed along the two generators that pass through it and, consequently, on the other edges which these generators cut.

This means that a curved edge can be totally free of unbalanced edge stresses and consequently does not require any rib or additional reinforcement of any type, giving rise thereby to structures of very light appearance.

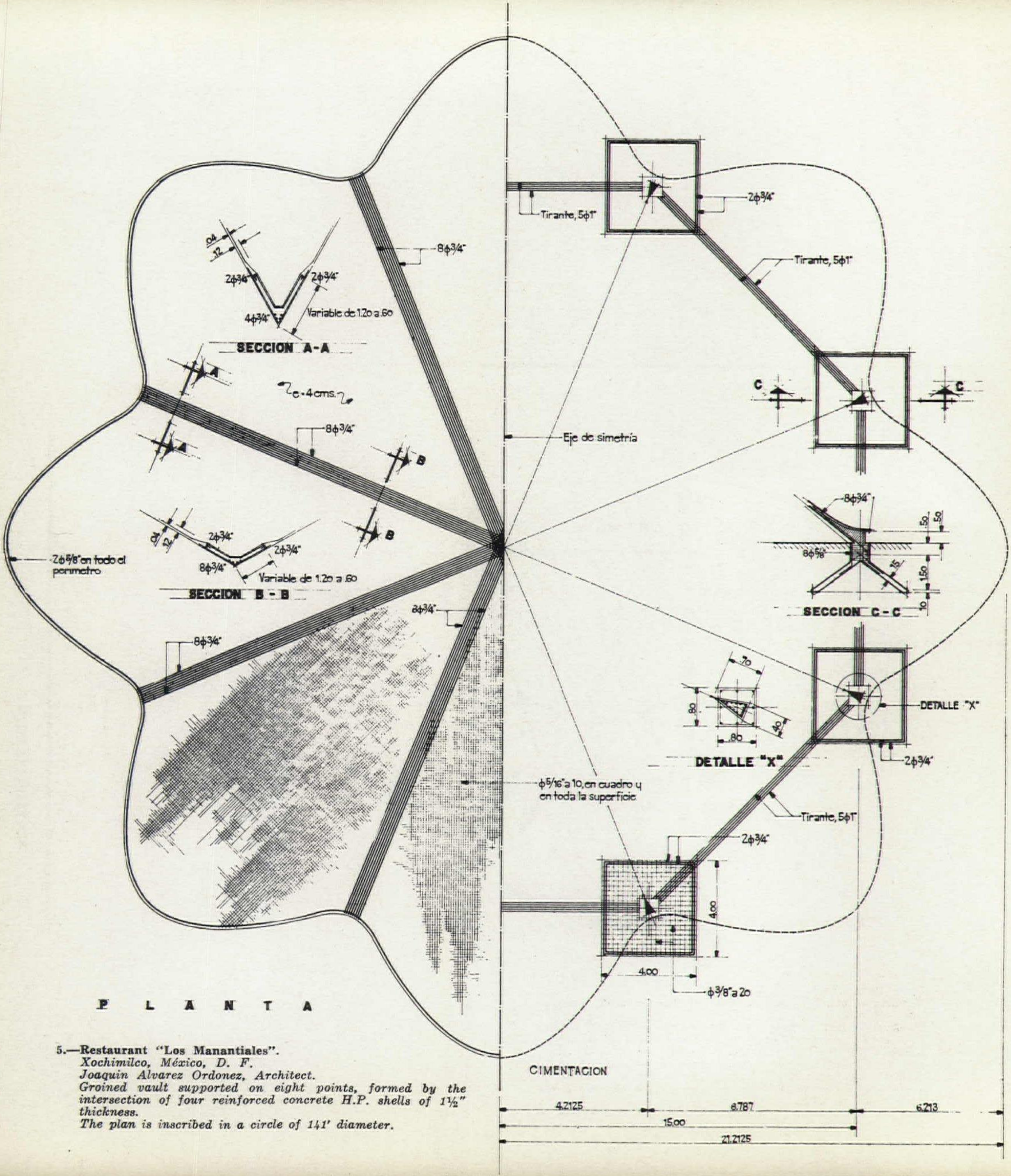
However, structures formed by a single paraboloid surface, either if they are limited by straight or curved edges, need to have part of these edges with complete supports since one of the two ends of each generator has to be suitably supported by an element capable of resisting the force which the former transmits. Considerations of symmetry in structures composed of several surfaces allow achieving free edges (without supports or edge ribs on the whole perimeter, provided that internal intersection edges having two contiguous surfaces are available. In these arists or groins are concentrated the edge stresses, and when symmetry exists, they produce forces contained in the vertical plane of the arist which can be resisted by its working as an arch or a beam.



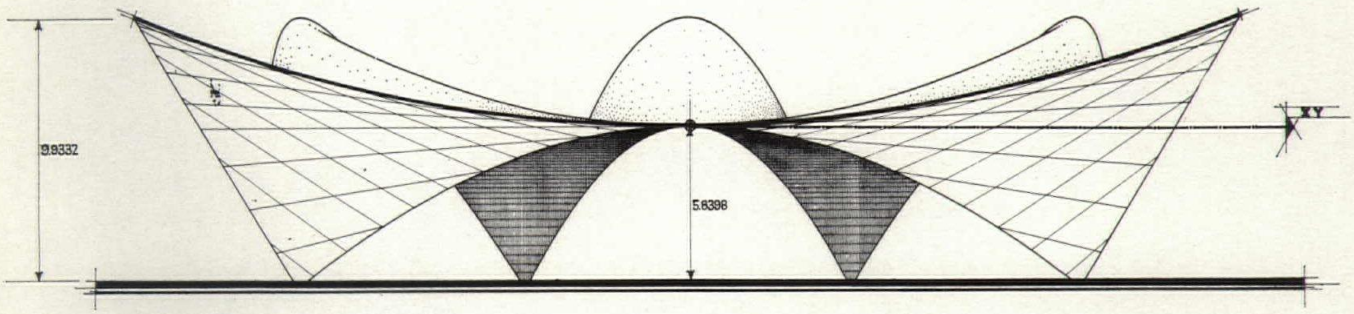
1.—Roof for a bandstand.
Low cost housing project in Santa Fe. México, D. F.
Mario Pani, Architect.
Fanshaped cantilever composed by reinforced concrete hyperbolic paraboloidical shells, 1½" thick.
40 ft. cantilever. Length of the outer edge, 60 ft.



6.—Open Chapel.
Residential development. "Lomas de Cuernavaca", Cuernavaca, México.
Guillermo Rosell & Manuel La Rosa, Architects. Roof formed by a single reinforced concrete H.P. shell of 1½" thickness, limited by: 1.—The intersection with the ground along two hyperbolas; 2.—The intersection with an inclined plane, giving a hyperbola along the big open facade; 3.—The intersection with two vertical planes, giving parabolas along the rear small opening.
The big facade opening has a span of 103' and a rise of 70'.



5.—**Restaurant "Los Manantiales".**
Xochimilco, México, D. F.
Joaquin Alvarez Ordonez, Architect.
Groined vault supported on eight points, formed by the intersection of four reinforced concrete H.P. shells of 1½" thickness.
The plan is inscribed in a circle of 141' diameter.



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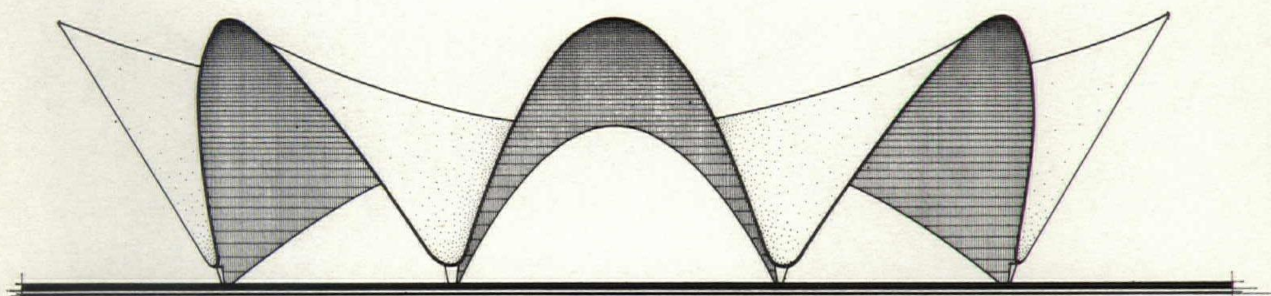
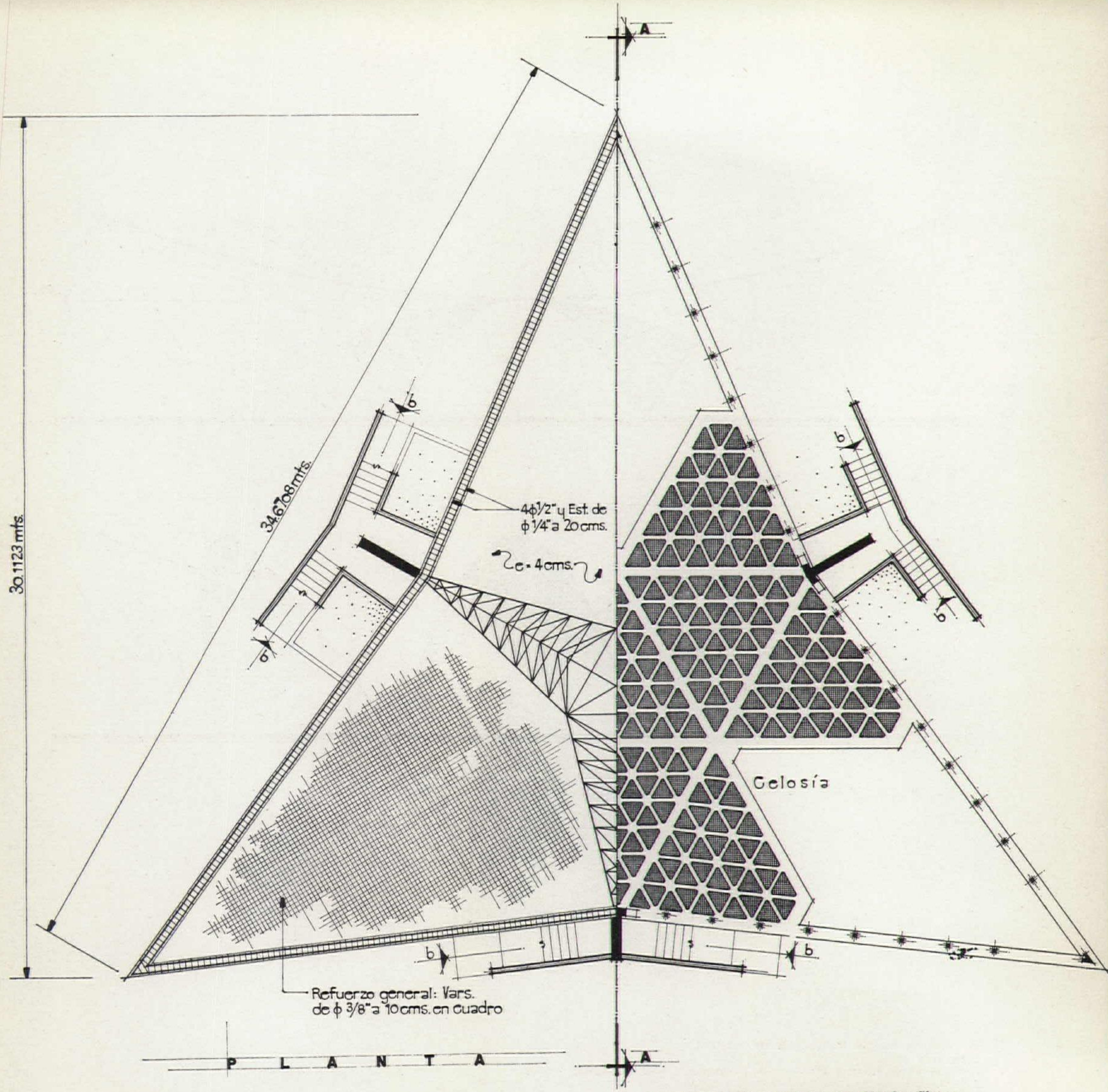


F A C H A D A B - B

7.—St. Vincent Chapel for the Charity Sisters.
Coyoacan, México, D. F.
Enrique de la Mora y Palomar & Fernando Lopez Carmona, Architects.
Structure formed by three reinforced concrete Hypar shells of 1½" thickness, limited by straight generators.
Triangular plan of 116' side.



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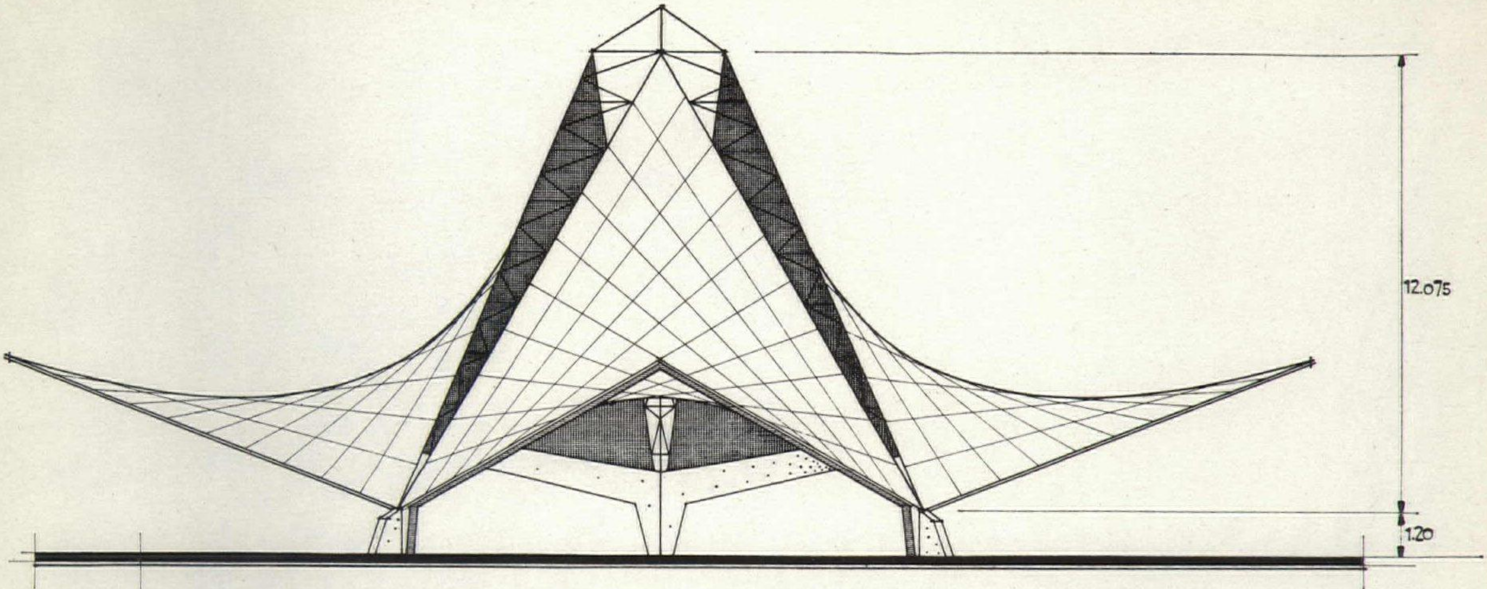

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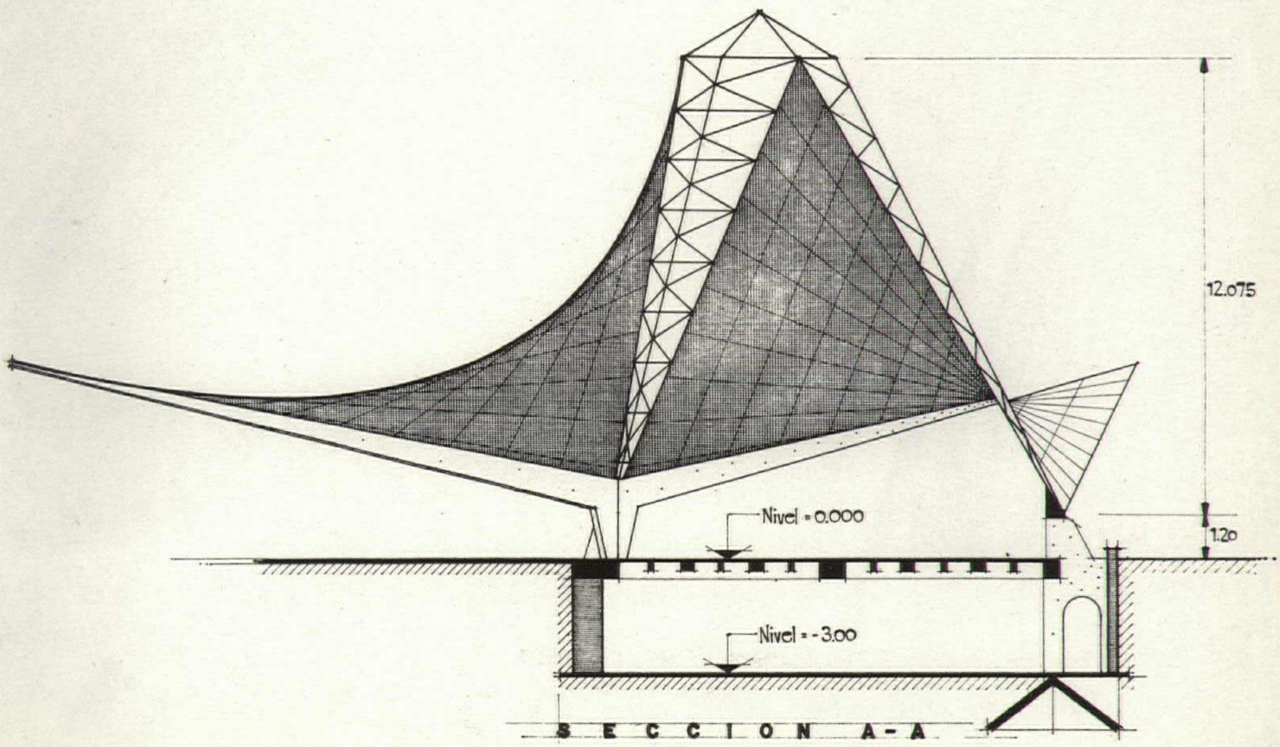
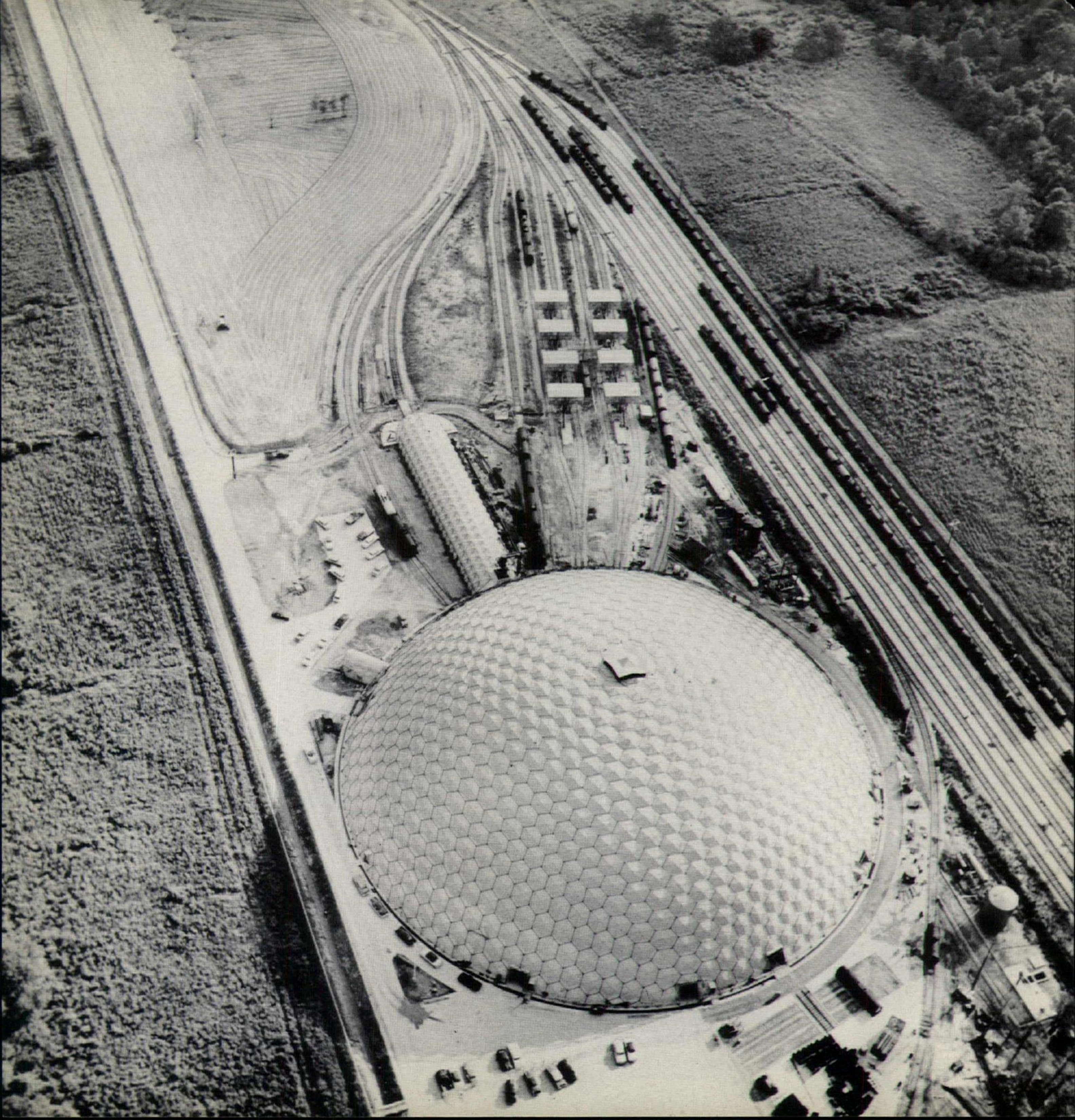


S E C C I O N A-A



THE DESIGN OF A GEODESIC DOME

M. E. UYANIK, *engineer and professor in the School of Engineering, N. C. State College, was intimately connected as engineering consultant with the design of the Baton Rouge dome.*

In deciding to use a spherical dome to roof a large circular area, the designer must consider the following factors which would affect the geometry of the dome:

THE SPAN

Unlike many other structural systems which might be chosen, the span of a dome is virtually a function of the architectural requirements alone. If a flat dome is chosen, the vertical clearance at the boundaries may become too small and render the periphery of the interior space useless. If a high section of a dome is chosen, excessive volume enclosure may render the structure too inefficient in terms of heating, light, etc. The first difficulty can be eliminated if the tension ring (which is necessary when the dome is less than a hemisphere) is raised and set on continuous walls or closely spaced columns. The second difficulty can only be eliminated by careful choice of proportion—this choice not being chiefly dependent on structural considerations.

Theoretically there is no limit to the span of a dome. But, as in any other structural framing system, there are practical limitations depending on the composition of the dome and the materials of which it is made. Concrete shells of up to 725 ft. span have been designed and are in construction at the time of this writing. A framed spherical dome of this span or larger can be considered a perfectly feasible structure with our present methods of construction.

HEIGHT

Once the architectural requirements have determined the necessary span, we can see that there are infinite numbers of spherical domes which could be placed on the span. To define the particular sphere on which the dome will be based, we will need to establish some third point on the dome which would in turn establish the height or zenith.

The radius of such a dome is calculated in terms of its height and span by the following simple expression: (See Fig. 1)

$$R = \frac{L^2 + 4H^2}{8H}$$

Where

R = Radius of the dome in feet

L = Span of the dome (or diameter of base circle) in feet

H = Height of the dome in feet

The span L is the dimension that is generally given. By choosing one of the other two dimensions the third one can easily be calculated. The height of the dome is a function of the economy in structural design as well as operational expenses after the structure is completed. A dome of lower height will have less roofing surface (surface area of a dome, $A = 2\pi RH$) and less volume inside to be heated and lighted (Volume $V = \pi H^2 \left[R - \frac{H}{3}\right]$. However, a flat dome will require more material in its tension ring than would a higher dome. In initial cost this extra expense may be equal to or greater than the savings realized in the smaller surface area of the flatter dome.

STRUCTURAL BEHAVIOR

Like all shell structures, if the thickness is very small compared to its span, the bending stresses occurring in the dome will become negligible, rendering the structure into an axially loaded system. This behavior of shells can be attributed to rather flexible action of shells under uniform or near uniformly distributed loads. The shell, due to its flexibility, will deform, reducing the bending stresses and adapting its shape to the pressure line of the loading thus resisting the loads through axial stresses. This behavior in shell structures is called membrane action and resulting stresses are called membrane stresses.

It must be remembered that the shell must have complete freedom of deformation. At supports, unless this freedom for deformation is provided, the shell will be subject to high, rather localized, bending stresses at its boundaries. If these bending stresses are eliminated by roller type supports, the dome reactions at supports will always remain tengential to the surface of the dome. When the tangent to the dome is other than vertical (or the dome is less or more than a hemisphere) a ring of tension or compression will be required to resist the horizontal components of the reactions (See Fig. 2). The vertical components will either have to be resisted by continuous supports or, if the support is not continuous, (such as a circular beam supported by vertical columns, or frame structures of some type) the analysis of stresses near the supports will become much more complex and require careful attention.

Loads on domes used as roofs are generally taken as the weight of the dome (dead load, D.L.), snow or ice (live loads, L.L.) and wind loads. All these loads are either uniform (equal per unit surface area of the dome) or variable as a function of the angle that the point (at which the load is acting) on the dome makes with the axis of revolution (Zenith Axis). All dead or live loads are considered as equivalent gravity loads acting vertically.
The following is a derivation of the membrane stresses $N\phi$ (meridianal) and $N\theta$ (circumferential) (See Fig. 3) for a uniform load of w per square foot of dome surface at some intermediate plane.

$dA = 2\pi R_o ds$ where $ds = Rd\phi$ and
$R_o = R \sin \phi$,
by substitution
$dA = 2\pi R^2 \sin \phi \, d\phi$

The weight of the dome (W) at any given zenith angle of ϕ:

$$W = \int_o^\phi w dA = \int_o^\phi 2\pi w R^2 \sin \phi d\phi = 2\pi w R^2 (1 - \cos\phi)$$

Where w = unit weight per sq. ft. of dome surface

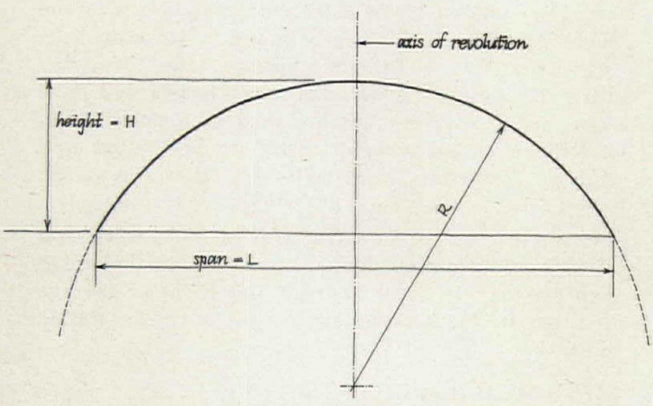


FIGURE 1

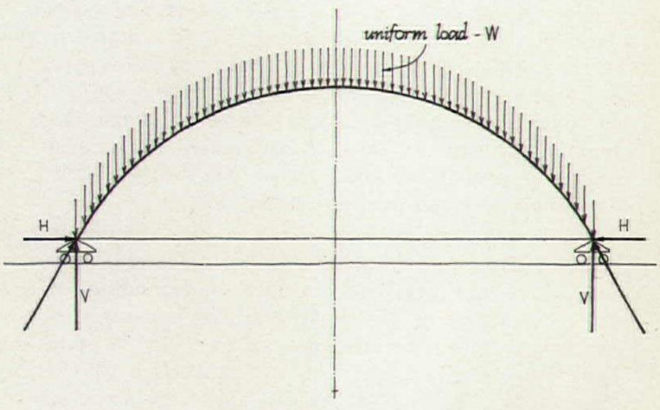


FIGURE 2

The vertical components of the reaction at its base (of angle ϕ):

$$V = 2\pi R_o N\phi \sin\phi = 2\pi R N\phi \sin^2\phi$$

For equilibrium $V = W$ or $N\phi \sin^2\phi = wR\ (1-\cos\phi)$

$$\text{or } N\phi = wR \frac{(1-\cos\phi)}{\sin^2\phi} \text{ compression when positive.}$$

In order to derive the equation for membrane stress $N\theta$ (hoop stress) the differential element ABCD in Fig. 3 is shown at a larger scale in the upper part of the figure. The forces acting on the element in Z direction are as follows: Components for forces acting normal to the sides AB and CD are:

$$(N\phi + \frac{\partial N\phi}{\partial\phi}\partial\phi)\ R_o\partial\phi\partial o$$

$$= R_o N\phi\partial\phi\partial o + R_o \frac{\partial n\phi}{\partial\phi} d\partial\phi^2 do$$

The second term can be neglected (since it is a quantity of second order differential). The component then will be:

$$R_o N\phi\partial\theta \qquad (a)$$

The component (in Z direction) of forces acting normal to sides AC and BD is: $N\theta R\partial\theta\partial\phi$ in the plane of the parallel circle of radius R_o. The component in the direction of Z-axis will be $N\theta R\sin\phi\partial\phi\partial\theta = N\theta R_o\partial\phi\partial\theta$ (b)

Finally the Z-component of external force w acting on the surface ABCD is: $ZRR_o\partial\phi\partial\theta = wRR_o\cos\phi\partial\phi\partial\theta$ (c)

By $\Sigma F_z = 0$ Expression (c) must be equal to the sum of (a) and (b). Thus $R_o N\phi\partial\phi\partial\theta + N\theta R_o\partial\phi\partial\theta = {}_wRR_o\cos\phi\partial\phi\partial\theta$

or

$$N\phi + N\theta = {}_wR\cos\phi$$

and

$$N\theta = {}_wR\cos\phi - N\phi \text{ where } N\theta = {}_wR \frac{1-\cos\phi}{\sin^2\phi}$$

$$\text{by substitution: } N\phi = \frac{(1-\cos\phi)}{\sin^2\phi}$$

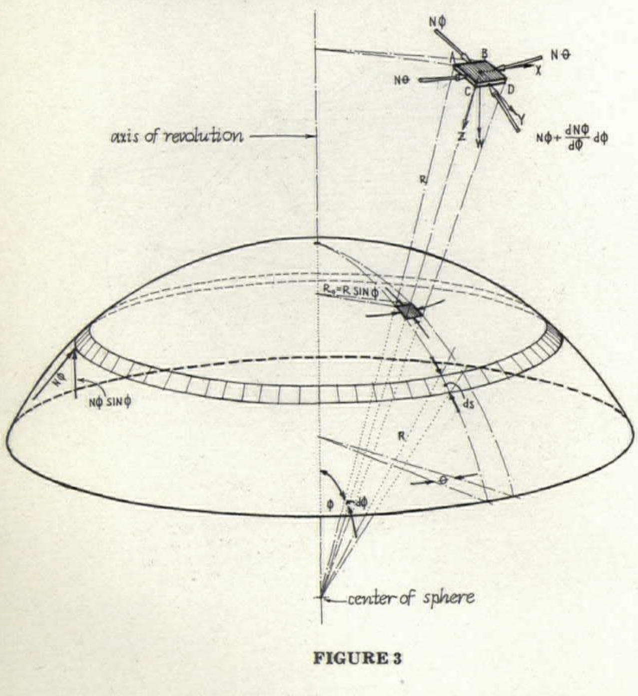


FIGURE 3

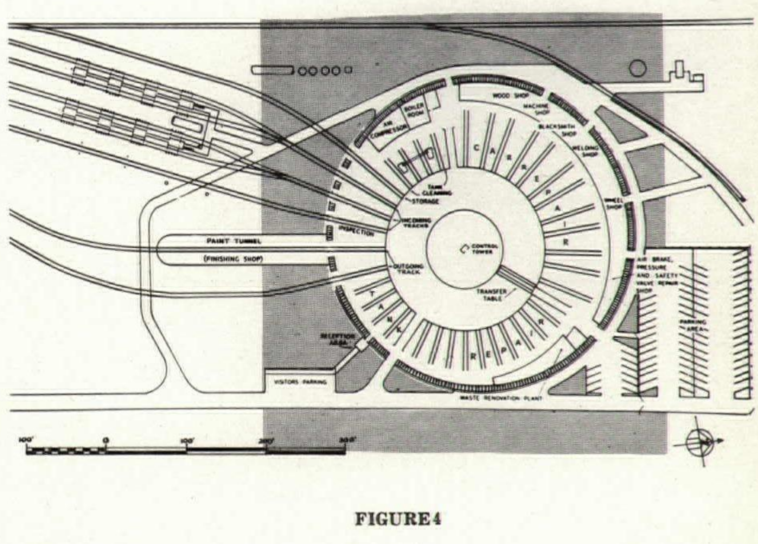


FIGURE 4

These stresses (per unit length) meridianal $N\phi$ and hoop stress $N\theta$ are primary membrane stresses acting in the shell of the dome when subjected to a uniform weight of W on its entire surface (symmetrical loading). For variable or unsymmetrical loads, such as wind or partial snow loads more rigorous stress analysis would be required (See "Plates and Shells" by Timoshenko.)

The thickness of the shell cannot be reduced so much as to cause the shell to buckle. The critical thickness is a function of the stress in the shell, modulus of elasticity, and Poisson's ratio of the material of which the shell is made. The following formula will give critical stress for a given thickness (See "Theory of Elastic Stability" by Timoshenko.

$$\delta_{cr} = \frac{Et}{R\sqrt{3\,(1-\mu^2)}}$$

Where δ_{cr} = critical stress in lbs/in²

E = modulus elasticity of the material of which the shell is made in lbs/in²

t = critical buckling thickness of material in inches

R = Radius of the dome in inches

μ = Poisson's ratio

FRAMED GEODESIC DOMES

Up to the present point nothing has been said about ways of establishing the required flexible rigidity of this type of dome.

The foregoing equation for critical buckling thickness does not directly apply to this problem. That this is so stems from the fact that domes of only one layer of shell surface would have to be thicker than a truss—framed dome having greater depth of radial section. The depth of trussing increases the stiffness with less material. The optimum depth versus material ratio is a complex problem which we do not propose to go into in this paper.

ENGINEERING FEATURES OF BATON ROUGE DOME

A Geodesic dome has recently been erected by the Union Tank Car Company of Chicago to roof their Maintenance Plant in Baton Rouge, Louisiana.

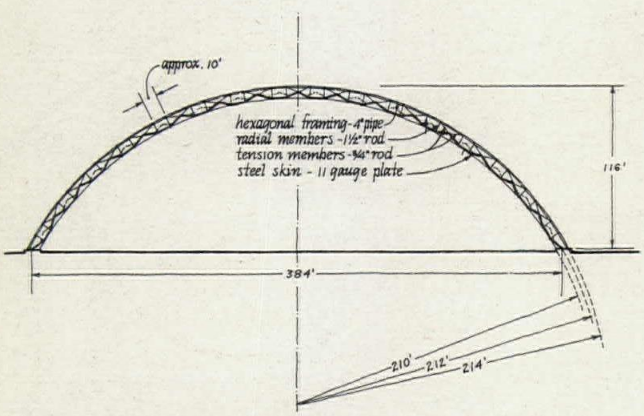


FIGURE 5

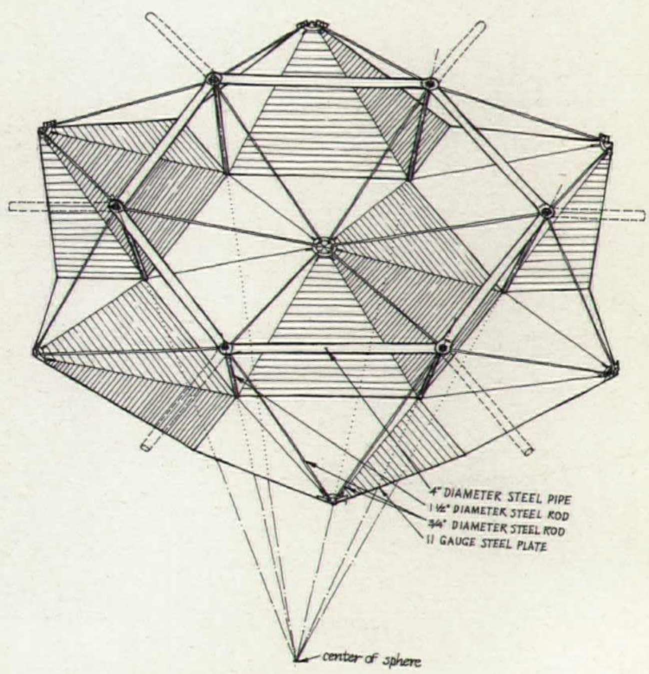


FIGURE 6

In this building they maintain a portion of their fleet of railroad tank cars which are leased to various industries throughout this country and Canada. A plan of this building showing this operation is shown in Fig. 4. Here is a building which effectively streamlines their operation and reduces the length of tracking as compared to a rectangular building. It was an ideal place for a dome since the climatic conditions in Louisiana do not require any heating of the rather large volume of air contained in the dome.

There have been various articles concerning this structure in trade and business magazines, as well as professional publications. None of these articles contain the engineering features which are outlined in this paper. The writer, in cooperation with the designers, Synergetics, Inc., of Raleigh, has served as consultant for the Union Tank Car Company on this project.

The dome is a unique structure insofar as its composition, method of construction, and size are concerned. The span of the dome is 384 feet and its height is approximately 116 feet. (See Fig. 5.) It is composed geometrically of three spherical surfaces of 214 feet, 212 feet, and 210 feet in radius.

The structural system is a so-called Geodesic grid (under the patent rights of Mr. R. Buckminster Fuller). It is composed of a hexagonal system of framing of 4-inch standard structural steel pipes, an involute skin of No. 11 gauge steel plate. These two surfaces (i.e., the pipes and the skin) are connected with a 1½ inch diameter solid bar positioned normal to the inner spherical surface of the dome. The radials have threaded upper ends, which serve as pins for the hexagonal pipe framing, with lower and upper nuts between which flattened ends of the 4 pipes are contained. A set of ¾ inch diameter rods (suspension rods) connect the apices of the involute skin plates with the vertices of the hexagonal pipe framing. A drawing of a typical panel used in the construction of the dome is shown in Fig. 6.

This system, as it can be noted, triangulates each of these three surfaces into a stable and efficient truss system. It will be further noted that there was no other roofing surface applied to the dome. The painted No. 11 gauge steel plate serves this purpose adequately. One of the reasons that the No. 11 gauge steel was used instead of a thinner gauge steel was the weldability of the thicker plate. Since the plate was used as a roof surface, it had to be

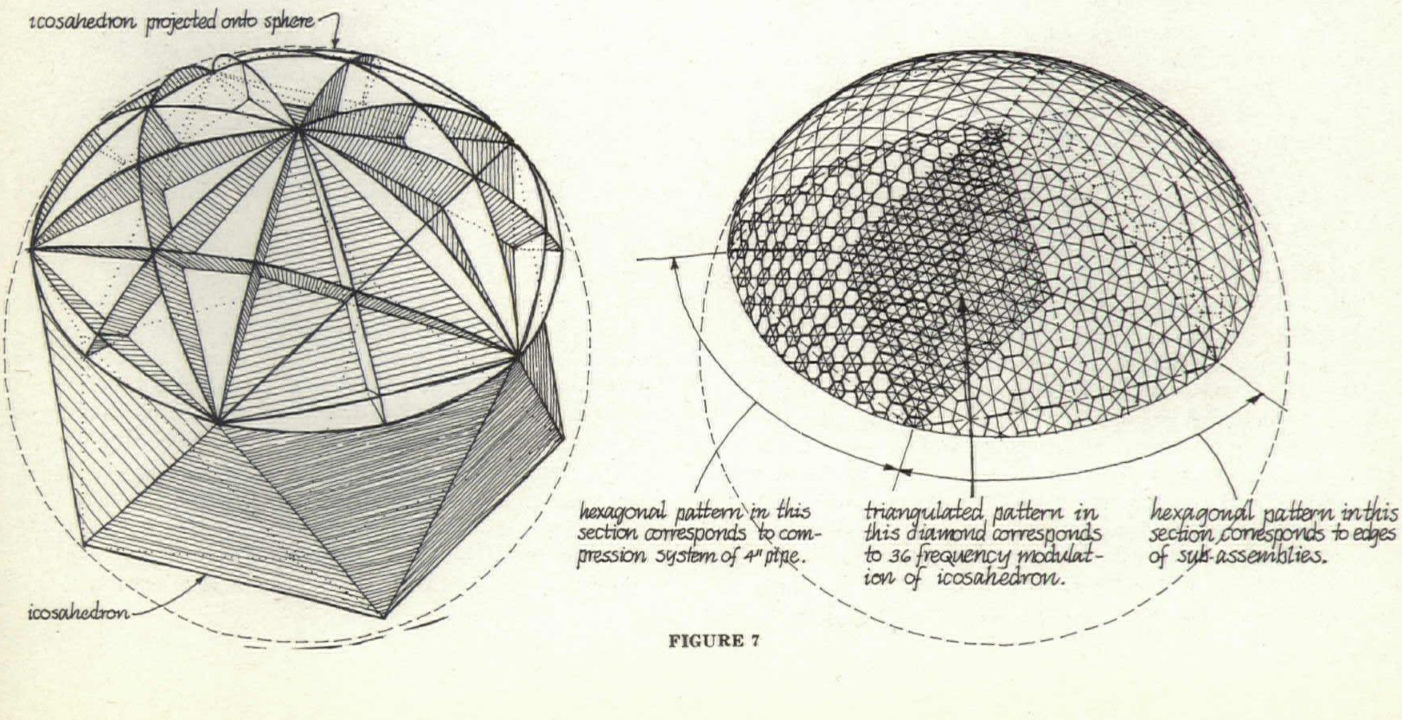


FIGURE 7

made waterproof by continuous welding around the boundaries of each panel. Thinner gauge metals are harder to weld and they are not as durable after they are welded.

GEODESIC SURFACE

This surface is a system of triangular (or multiples of triangular) shapes composing a spherical surface. The sides of each of the groups of triangles comprise chords on a great circle of the sphere; such a system, consisting of either individual members such as the pipes or stiffened edges of the surface, such as the skin, is a Geodesic system.

A spherical surface (or a portion thereof) can be subdivided into a number of equal spherical diamonds provided the geometry of the subdivision is based upon some regular polydedron. Further symmetrical subdivision of the spherical diamond can be made in such a way as to yield some minimum number of kinds of parts. This particular dome in Louisiana has a five-way symmetry based upon the icosahedron. The Geodesic system has the advantage of having the least amount of variation in the length of indivdual members of subdivision described above. The number of triangles on the long diagonal center line of the diamond will constitute the frequency number of the system. The dome in Baton Rouge was a 36 frequency dome. The length of each pipe composing the side of the hexagonal framing was approximately 10 feet. This length can be increased or decreased by increasing or decreasing the frequency number chosen for the geometry.

The Geodesic system is advantageous insofar as it keeps the sizes of these hexagonal framing members rather uniform and small compared to the span. This is fortuitous, due to the fact that the stresses in the framing system do not vary greatly from zenith to the base (See Fig. 8a and 8b); thus the use of one size member on the entire dome surface does not constitute an excessive burden of non-working material. Any other type of ribbed dome in which the rib members vary in length and are located in positions which would cause concentration of stresses will certainly be subjected to bending stress and consequently become less economical. Of all the framed domes, the action of the Geodesic dome is the nearest to membrane action (shell behavior). As it was previously pointed out, the most economical behavior of any roof structure of large span is that of the membrane, carrying stresses axially rather than through bending combined with axial stresses.

DESIGN LOADS OF BATON ROUGE DOME

Dead Load

A dead load of 8.4 lb. p.s.f. on the dome surface was assumed to be constant for the entire surface. The components of the load w_d (referring to axis shown in Fig. 3) were as follows:

$w_x = 0$

$W_y = \sin \phi$

$w_z = w_d \cos \phi$

Live Loads

A live load (w_L) of 20 lb. p.s.f. as required by the State of Louisiana Code was used with the following components:

$w_x = 0$

$w_y = w_L \sin \phi \cos \phi$

$w_z = w_L \cos^2 \phi$

An ice load of ½ the live load (10 lbs. p.s.f.) was used in combination with wind loads.

Wind loads

Although the Louisiana State Code does not require as much as 36 lb. p.s.f., this pressure was used as design pressure for wind. The reason for the use of higher wind pressure stemmed from the fact that frequent hurricane winds in the last few years have repeatedly reached velocities of 100 miles per hour and over in this area of the United States. (36 lb. p.s.f. pressure is equivalent approximately to 125 mile wind velocity.)

Wind pressure (and suction) was assumed to vary as follows:

$w_x = 0$

$w_y = 0$

$w_z = w_w \sin \phi \cos \phi$

Where w_w is the wind pressure and angle ϕ is the angle that the element on the dome surface (on which the pressure is considered) makes with the horizontal axis of the dome in the direction of the wind.

After the stresses were determined, design of the dome was made for the following two load combinations: (See Figs. 8a and 8b).

Combination I — Dead Load, full live load

Combination II — Dead load, Ice (½ live load)
Wind loads

ANALYSIS

The following assumptions in the analysis of the dome were made:

The primary stresses were assumed to be carried by the hexagonal (4" diam. pipe) framing with all its corners stabilized by 11 gauge steel plate, radials (1½" diam. solid rod) and a ¾" diam. rod constituting a stable triangulation of every intersecting point of the pipes. The pipes were hinged at their points of intersection by two 1½" diam. nuts, one at the bottom of the stack of flattened ends of the three pipes and one at the top, with the threaded top end of the radial constituting the pin at these points. Further, the ends of the ¾" diam. tension rods were held by a plate inserted between the lower nut and the stack of pipe ends.

The dome was assumed to be of sufficient flexibility to be under membrane stresses only. The sheet metal surface was assumed to be the main surface of loading of the dome subject primarily to tensile stress. The 1½" radials were assumed to be acting as shear struts between the 11 gauge skin and the pipe frame for localized bending moments caused by overloading. The ¾" tension rods were assumed to be primarily suspension rods to transmit the loading of the steel plate surface (to which they were welded at the apices of the foldings of the steel plate) and the hubs of pipe framing to which they were connected.

STRESSES

As described above, membrane stress analogy was used in finding the stresses in the surface of the dome. Load Combinations I and II are presented in Figures 8a and 8b.

The members were designed for maximum possible load combination throughout the entire surface of the dome, with the assumption that the path of stress was defined by the configuration of the hexagonal pipe framing.

Needless to say, the governing design stress was Combination II, (dead load + ice + wind).

There are five main door openings on the east side of the dome with five railroad tracks leading into the dome. These doors had sufficient clearance to satisfy the A.R.E.A. Specifications for Clearances. The framing of these doors was designed in an inclined plane tangent to the dome surface at their

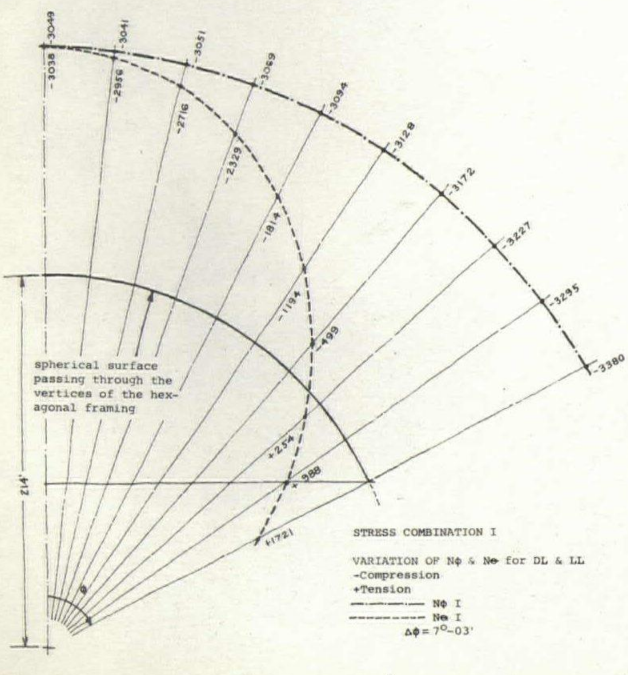


FIGURE 8a

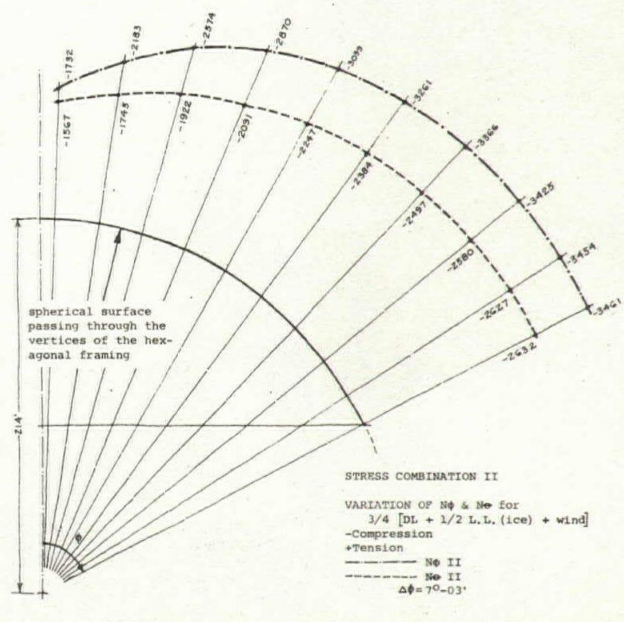


FIGURE 8b

tops, providing continuity of support around the interrupted boundary of the dome. There were other small doors, about nine in number, for delivery trucks. Only two of these required special framing.

TESTS

The maximum length for any 4″ pipe in the hexagon was approximately 12 feet. Since it was decided to flatten the ends of the pipe to provide pin holes, the behavior of this flattened part was investigated by full size tests in column action and was found to be completely stable in the development of the full strength of the column. A panel, as shown in Fig. 6, was built to full scale in the Chicago plant of the Union Tank Car Company and tested for possible handling stresses that were anticipated during erection. The behavior of the panels during the erection in the dome has substantiated the satisfactory results of this test.

FABRICATION

There were only seven items of material which constituted the steel parts of the structure above its base assemblies, namely: 11 gauge sheet steel, 4 inch pipe, 1½ inch and ¾ inch rods, 2 inch sleeves to provide additional bearing at pinned joints, and 1½ inch nuts and washers.

Panels similar to those shown in Fig. 6 (321 in number) were fabricated on the site on jigs made by the Union Tank Car Company. A maximum tolerance of 1/16 of an inch was observed in the manufacture of these panels. Keeping the tolerances to a minimum was necessary for the procedure of erection which was used since no job fitting was permitted on a panel when placed in position on the dome. This close control of geometry was of utmost importance in completing the erection.

FOUNDATION

The dome is supported on a continuous tension ring—grade beam combination of a size of 4 feet by 2 ft. 6″. This beam contains sufficient continuous steel to develop 600,000 lbs. tension. This ring beam rests on 81 bell bottom piers of 18 inch diam. shaft expanding into 4′ 6″ cone one foot above its base. These piers are moderately reinforced and rest on a stiff clay strata 14 ft. below the present grade. The tension ring grade beam had additional reinforcing steel for bending and torsional stresses produced by loads on the dome acting on a curved beam supported by piers approximately 15 feet on center along the periphery. Since the tension ring was embedded in a mass of concrete of 2½ x 4 ft., the tension slack in the ring due to dead load was permitted to take place and its effect was found to be very small. The hexagonal framing was hinged to the base to give freedom of rotation to the boundaries of the dome. However, no freedom of linear movement was permitted. The effect of this restraint was taken into account in the design of the lower members.

ERECTION

The dome was erected with the lower ring of panels being placed first and secured to their supports and welded to the adjacent panels around the entire periphery before the second ring of panels were placed. This process of welding all edges was observed for all panels around each ring. This procedure was continued well above the main door openings (about 20 ft. in height) without any particular shoring of the panels. The action of the panels during this part of the erection constituted compressive stresses in the interior steel plate. However, the intensity of these stress was apparently small, since the plate resisted these compressive stresses with very little localized bucklings. However, as the erection continued, the panels became necessarily more horizontal in their positions, producing higher compressive stresses and requiring some scattered shoring of the panels in place until all the ring panels at that elevation were completed. The above mentioned localized bucklings in the plate did not, however, cause any difficulty in completing the theoretical spherical surface. The deflection of the zenith of the dome added to localized distortions in the plate resulted in less than 0.3 of one percent error in the vertical height of the dome. This was, as mentioned before, due to close control of the manufacturing tolerances and strictly adhering to proper fitting of each panel to the adjacent panels without using field fitting processes such as welding torches or welding machines.

This remarkable structure, costing less than $10.00 per sq. ft. is a cooperative achievement of various organizations and people. I believe the Union Tank Car Company which owns the structure, having manufactured the panels with its own crews, is to be congratulated. Synergetics, Inc., of Raleigh, headed by James Fitzgibbon, deserves a great deal of credit for the details and close supervision of the entire project. Professor Duncan Stuart of the School of Design, responsible for the geometry of the structure, deserves a great deal of credit for the excellence of his work. Mr. T. C. Howard and Mr. David Sides, Jr., of Synergetics, Inc., with their brilliant ideas of detailing of various parts of the dome should be recognized. Mr. Pete Barnwell of Synergetics, Inc., deserves a great deal of credit for his on the job supervisory work. Nichols Construction Company, who erected the dome, and Battey, Childs, Engineers, of Chicago, who supervised the project, can take pride in their work which was so well performed. Further credit should be extended to Professor Richard Lewontin for his excellent work in programming the major mathematical work into the IBM computer.

In conclusion the writer believes that the premanufactured panel type dome, such as the Geodesic dome, is one of the most versatile methods and certainly an economical way of spanning large spaces. The membrane action in large spans constitutes a most economical method of using material if erection costs could be kept low. From the present vantage point, one can see that much can be improved in the erection and composition of the various parts of this dome. Despite all of the difficulties that were encountered during erection, the structure is assumed to be one of the most economical of its kind.

The writer wishes to thank Professor D. R. Stuart for his great help in preparing this paper.

STATISTICS

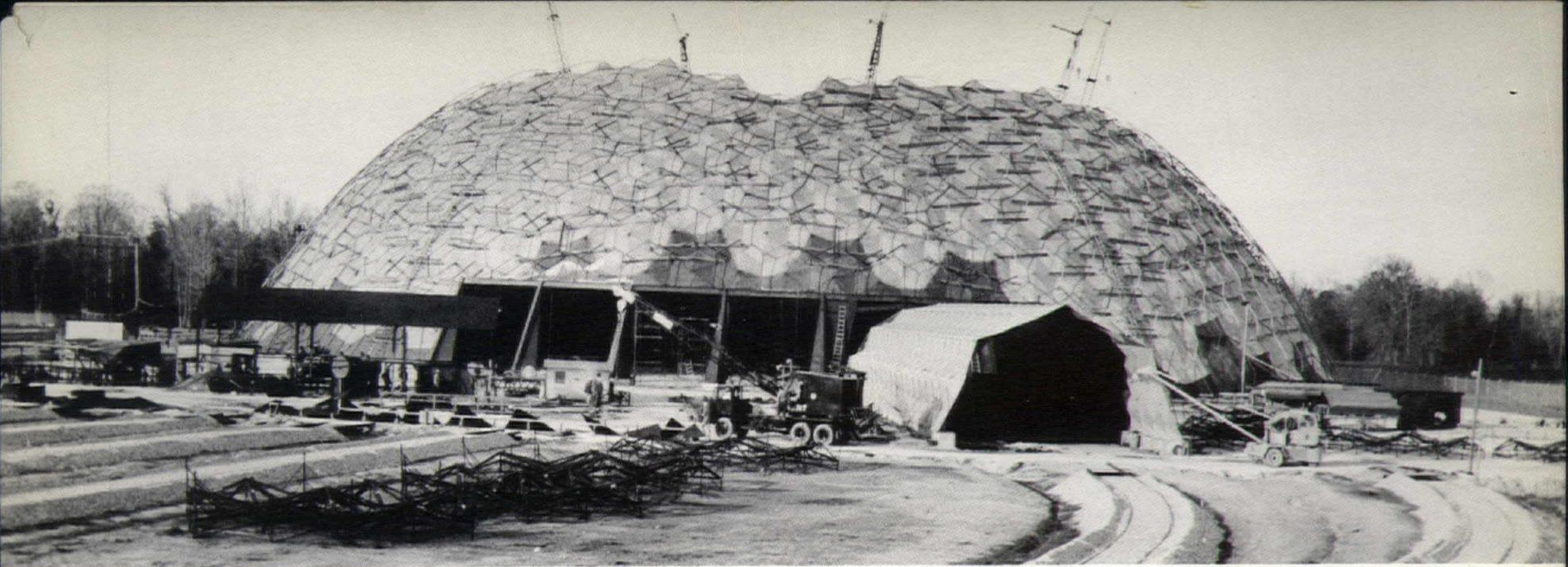
Diameter	384 feet
Diameter Ground Area	110,000 sq. ft.
Height at Zenith	120 feet
Total Surface Area of Panels	154,900 sq. ft.
Total Weight above Foundations	567 tons
Number of Steel Panels	320
Size of Paint Tunnel	
Length	200 feet
Height	20 feet
Width	40 feet
Total Ground Area	8,000 sq. ft.
Size of Interior Structure	
Diameter	100 feet
Height	80 feet
Area Encompassed	7,500 sq. ft.

Geodesic Dome Design	Synergetics, Inc. Raleigh, N. C.
Dome Structural Consultant	Dr. M. E. Uyanik Raleigh, N. C.
Architects & Engineers	Battey & Childs Chicago, Ill.
Field Superintendent	Lloyd Botts Chicago, Ill.
General Contractors	Nichols Construction Co. Baton Rouge, La.
Lighting Consultant	Abe Fedders
Color Consultant	George Holt

The union dome is approximately a one-quarter sphere, designed and constructed of steel as a three dimensional curved truss. The truss, approximately four feet deep, is designed as a unit cell system of octehedra in which the involuted 11 gage sheet steel surface material is employed both as the weathering surface and as the inner member tension system of the truss. The outer hexagonal array is composed of 4 inch steel pipe sections approximately 9 feet long positioned with ¾ inch tension rods. The compression pipe elements and the tension rods lie along the typical geodesic great circle grid lines. The steel sheet fold lines repeat the geometry of the pipe and rod systems.

The sheet was cut in diamond (two triangles) shapes and set in jigs on the ground. Nine diamonds made up a typical sub-assembly, 320 sub-assembly panels total. The tube and rod truss elements were then set in position, on the jigged sheet diamonds, the sheet edges were butt welded and the sub-assembly moved to the sand blast and paint area, and then ground stored ready for assembly into the dome. The sub-assemblies, weighing about 4,000 pounds, were handled on the ground with a cherry picker crane and lifted into place with mobile crane rigs.

Construction started from the ground up; the sub-assembly units were positioned and welded into place. The geodesic dome is self scaffolding and climbable at all times. Pipe 'push-pull' poles were used to make small contact adjustments between sub-assemblies during construction. Erection time was exactly five months total including an estimated four weeks lost time for weather and strike shutdown.



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